Incremental Airline Schedule Design

by

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B.Eng. Civil Engineering
Chulalongkorn University, 1995

Submitted to the Department of Civil and Environmental Engineering
and Operations Research Center
in partial fulfillment of the requirements for the degrees of

Master of Science in Transportation

and

Master of Science in Operations Research

at the
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
September 1998
[February 1999]

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Abstract

We consider the problem of integrating flight schedule design and fleet assignment decisions at airlines. The flight schedule design problem involves selecting and scheduling the set of flight legs that an airline will include in its service network. Fleet assignment involves assigning a particular aircraft type to each flight leg in the schedule. Due to the particularly challenging nature of schedule design problems, we limit our focus to that of *incremental* schedule design. Incremental schedule design involves the modification of a given flight schedule to produce an improved schedule by adding, deleting, and rescheduling flight legs. We present models and algorithms to achieve incremental schedule design and unlike previous schedule design efforts, we explicitly model flight demand and supply interactions.

We present two case studies, using our models and algorithms. The first case study allows flight additions and deletions, while the second allows flights to be rescheduled. Future case studies well integrate these flight modification options. In our first case study, high-yield flights are maintained in the schedule and low-yield flights are dropped. Although the resulting schedule incurs higher spill costs, the savings from flight operating costs are sufficiently large to offset these higher spill costs, resulting in a more profitable schedule. The second case study, allowing flights to be rescheduled, considers several network sizes including the domestic network of a large U.S. airline. We consider *Free Flight*, a system allowing reduced flying times due to improved utilization of the national airspace. We find that reductions in flying times of about 10% can lead to dramatic cost savings for the airline, including reductions in the number of aircraft needed to fly the flight schedule.

Thesis Supervisor: Cynthia Barnhart
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Acknowledgments

I would first like to thank Professor Cynthia Barnhart for being a remarkable source of inspiration, a role model, a mentor, and a friend. I thank her for her tireless effort in making this thesis a readable document.

I am grateful to the Ananda Mahidol Foundation who provides me a wonderful opportunity to pursue my higher education here at MIT. Thanks are also due to faculties at the Department of Civil Engineering, Chulalongkorn University.

I wish to thank Professor Moshe Ben-Akiva, Professor Joseph Sussman, and Professor Nigel Wilson for their valuable suggestions. I would also like to thank Brian Rexing, Tim Kniker, Andy Armacost, and Niranjan Krishnan for their technical supports on this research. Many thanks go to people at United Airlines with whom I worked closely, notably, Chris Spidle, Steve Morley, and Raj Sivakumar.

I would like to thank my friends at the Center for Transportation Studies, especially, Hitoshi Ono, Dave Cuneo, Joan Walker, Hong Jin, Maria Marangiello, and Sydney Miller.

Many thanks are due to my best friends, Ka, Nog, and Nick. I thank them for being there when I need them most. Special thanks to Jad, Boy, Berm, Vijak, Kaeo, It, P'Sao, and Chalee.

Finally, I would like to thank the most important people in my life—my family. Their support, love, and blessing are most precious and invaluable to me. I love them all.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 Overview

In scheduled passenger air transportation, airline profitability is critically influenced by the airline's ability to construct flight schedules containing flights at desirable times in markets (defined by origins and destinations pairs) with high demand. To construct such schedules, airlines engage in a complex decision making process referred to as *airline schedule planning.*

The airline schedule planning process is comprised of the four major steps shown in figure 1-1. In the first step, *schedule design*, the planners decide when and where flights should be flown. The design is subject to several restrictions, which must be verified as the preliminary schedule is processed in the next steps. In the second step, *fleet assignment*, every flight in the schedule is assigned to an aircraft type, called a fleet, subject to the aircraft types and numbers available. The objective is to assign aircraft types to flights such that the passenger demand and aircraft size (in terms of the number of seats) are matched as well as possible. Once the schedule has been fleeted, the next step is to determine the *aircraft rotations* (a sequence of flights beginning and ending at the same location) for every plane subject to maintenance requirements. This step is referred to as *aircraft routing.* In the last step, *crew scheduling*, the planners allocate *flight crews*—pilots and flight attendants—to flight legs such that all work rules are satisfied, each flight has the necessary crew and crew costs are minimized.

Operations researchers and air transportation professionals have extensively studied the fleet assignment, aircraft routing and crew scheduling steps of the airline planning process and have achieved impressive results. However, the problem of schedule design is still

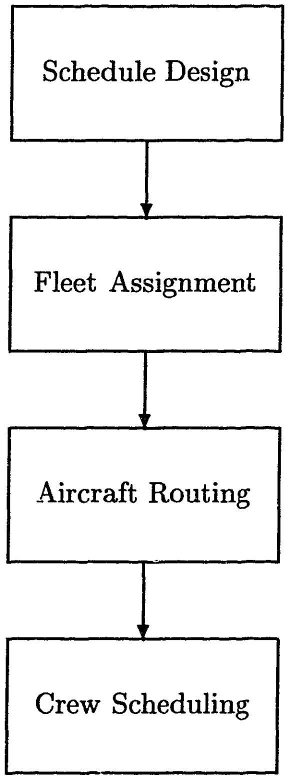


Figure 1-1: The Airline Schedule Planning Process.

relatively unexplored, and unsolved. One of the obstacles that prevents researchers from solving optimally the schedule design problem is its prohibitive size and complexity. In this thesis, we begin to explore the question of schedule design by focusing on a simplified version of the problem, that of incremental schedule design. Incremental schedule design involves a given flight schedule, passenger demand, aircraft of different types, and a set of flights, some of which might be flown. The objective is to select which flights will be flown (and which will not) and to assign feasibly an aircraft to each selected flight.

1.2 Airline Scheduling: An Overview

In this section we first review relevant literatures on the demand for air travel, the supply of air passenger services, and their interactions in the context of scheduled air passenger transportation. As will be seen, such understandings are essential for the development of an efficient flight schedule. Next, we give an overview of airline scheduling practice.

1.2.1 Scheduled Air Passenger Transportation: Demand and Supply

In order to be able to design an optimal schedule, knowledge of air passenger demand and supply for air transportation service, as well as their interactions, is essential. We present first the literature addressing the problem of demand forecasting and estimation. Then, for the supply side, we review literature on flight network structure and route generation. Throughout, we describe how demands and supplies are dependent on one another and we emphasize the resulting difficulties that arise in the airline schedule design process.

Demand

The demand for air travel (as well as other modes of transportation) is a derived demand [33]; it is derived from other needs of individuals. For example, the purpose of a person's trip might be to visit friends or relatives, or to attend a business meeting, rarely is it for the mere sake of traveling on a plane (car, bus, etc.).

Recall that a market is defined by an origin and destination pair. For example, Boston-Los Angeles is a market, and Los Angeles-Boston is another distinct market. These markets are referred to as *opposite markets*. Demands for a pair of markets do not interact unless the markets are *parallel markets*. Boston-San Francisco and Boston-Oakland are examples of parallel markets because San Francisco and Oakland are sufficiently close to each other that passengers are often indifferent between which airport they arrive. (Similarly, the opposites of these two markets, San Francisco-Boston and Oakland- Boston, are parallel markets.)

There are a couple of ways to estimate the *total market demand* for air travel. Teodorovic [37] details a methodology for estimating total air travel demand using a classical four-step transportation planning process, namely (Papacostas and Prevedouros [27]):

1. trip generation;
2. trip distribution;
3. modal split; and
4. trip assignment.

The objective of trip generation is to forecast demand for travel of each travel analysis zone or region. The purpose of trip distribution is to forecast demand for travel for each origin-destination pair. In modal split, origin-destination demands are categorized by mode, e.g., air, auto, transit. In trip assignment, the trips for each origin-destination pair are assigned to specific routes or itineraries.

Simpson [31] presents another way to generate projected demand between any two points (cities) using a modified gravity model based on the population in each city and the distance between the cities. The model has an additional multiplicative term to modify the share of air travel in light of competition from other modes of transportation, including auto, bus, and train.

For the purpose of schedule design for a given airline, we are interested in the *unconstrained market demand*, that is, the fraction of the total demand in a market, termed as *market shares*, that the airline is able to capture. Unconstrained market demand can be allocated to the itineraries in each market to determine *unconstrained itinerary demand.* The term *"unconstrained"* refers to the fact that the demands represent highest attainable demand levels for the airline (or total number of requests). The actual numbers of demand accommodated (or the total number of bookings) are therefore less than or equal to these unconstrained numbers.

Simpson and Belobaba [33] present several regression models that regress the unconstrained market demand for a carrier on a number of explanatory variables, e.g., market demographic variables, quality of service variables, fares, etc. The market demographics variables can include the population of the market's origin and destination, and the corresponding average disposable incomes, for example. The quality of service variables include, among other things, schedule reliability and total travel time. The total travel time can be further decomposed into two main elements, namely, out-of-vehicle travel time and in-vehicle travel time. The out-of-vehicle travel time is a function of the frequency of the services in the market and the in-vehicle travel time is a function of the aircraft cruise speed and the distance of the exact route that the flight takes. This model illustrates that demand is a function of supply, that is, the unconstrained market demand for a carrier is a function of its flight schedule (with frequency of service being one critical element). The *total* market demand can also change as a result of changes in the flight schedule. Specifically, additional demand can be *stimulated* [33], or more precisely diverted from other modes of transportation, when the number of itineraries/flights (i.e., supply) is increased (given that the demand has not yet reached the maximum demand) and vice versa. It is also true that supply is a function of demand: the carrier purposefully designs its schedule to capture the largest market shares.

Supply

The airline develops its flight network to compete for market share. The first step in developing the flight network is to adopt an appropriate network structure. Unlike other modes of transportation in which the routes are restricted by geography, most of the time, the route structures for air transportation are more flexible. Simpson and Belobaba [34] present three basic network structures, namely,

1. *a linear network*,

2. *a hub-and-spoke network*, and

3. *a point-to-point (complete) network*.

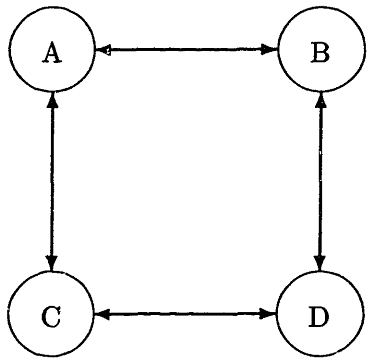
Figure 1-2 depicts these network structures for 4 locations.

Linear networks require a total of $2n$ links for network with n nodes, where each node corresponds to a city or an airport. This type of network is commonly used by railroads and urban transit systems where the routes are relatively limited. Hub- and-spoke networks require $2(n-1)$ links for networks of size n. This type of network has been adopted by most U.S. airlines since their deregulation in 1978 [40]. Its main advantage derives from connecting opportunities at the hub airport enabling airlines to consolidate demand from several markets onto each flight. Complete networks require $n(n-1)$ links for networks of size n, that is, they require one link for each market. This enables airlines to serve more markets especially when the demands in some markets do not warrant direct services. Simpson and Belobaba [34] also note that the hub-and-spoke network structure creates more stable demand at the flight leg level. Normally, there is some degree of variability (due to peak periods) in the demand for travel between any pair of origin-destination. By mixing and consolidating demands from different markets on each flight leg, the hub-and-spoke network can reduce variations in the number of passengers at the flight leg level, since markets have different demand distributions.

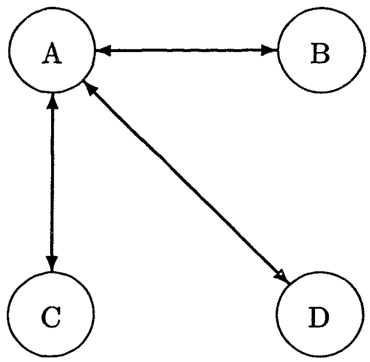
Demand and Supply Interaction

Hub-and-spoke networks illustrate demand and supply interactions. To see this, consider removing a flight leg from a *connecting bank or complex*. A bank or complex is a set of flights arriving or departing a hub at airport in some period of time. Banks typically are designed with a set of flight arrivals to the hub, followed by a sequence of departures from the hub to facilitate passenger connections. The removal of a flight from a hub can have serious ramifications on passengers in many markets through out the network. The issue is that the removed flight does not only carry local passengers from the flight's origin city to the flight's destination city, it also carries a significant number of passengers from many other markets that happen to have that flight leg on their itineraries. In the view of the passengers in those markets, since the frequency of service is decreased, the quality of service is deteriorated (to different extents from market to market, depending on the market shares that that particular itinerary previously carried). The result will be that the carrier will experience a decrease in its unconstrained market demands in the affected markets. The situation will be the opposite when a flight leg is added to the bank.

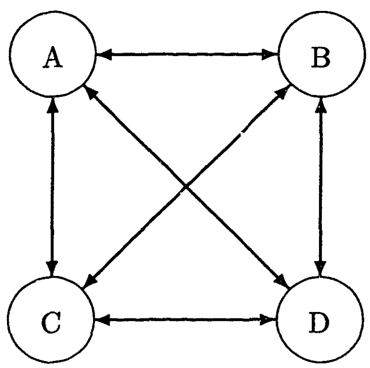
These demand and supply interactions are evident in all modes of transportation. However, the sensitivity of supply with changes in demands, and vice versa, differs from one mode to another. Within any mode of transportation, the services provided by different providers are highly substitutable, that is, the provided services of a given mode are not



(a)Linear Network



(b)Hub-and-Spoke Network



(c)Complete Network

Note: Two links are shown as one line with two arrow heads.

Figure 1-2: Basic Network Structures for $n = 4$ Cities and $n(n-1)$ Markets.

differentiable to any significant degree. A high degree of substitution reflects a high level of competition. However, substitution and competition exist only when there are many players in the market. This is evident in air passenger travel and inter-city bus travel, for example. For air transportation, Teodorovic and Krcmar- Nozic [38] show that flight frequencies and departure times are among the most important factors that determine passengers' choice of air carrier when there is a large number of carriers in the same market (i.e., high levels of competition). In most of the city public transportation systems, however, there are very few providers. This explains why there are relatively fewer interactions between demand and supply in most public transportation systems. Rao [28] provides a comprehensive review and an excellent framework for public transportation network planning.

1.2.2 Airline Scheduling Practice

Etschmaier and Mathaisel [18] provide a review of airline scheduling literature. They categorize the works related to airline scheduling into two families:

1. *schedule construction*, and

2. *schedule evaluation.*

Schedule Construction

The objective of *schedule construction* is to develop a schedule, defining an origin, a destination, a departure time, and an arrival time for each service to accommodate passenger demands given available resources. Constrained resources include vehicle, crew, maintenance facilities and staff, etc. Schedule construction is typically comprised of two sequential steps [15]:

1. frequency planning/optimization, and

2. vehicle planning/optimization.

This two-step schedule construction process is applicable to every mode of transportation. Note that this schedule construction paradigm is equivalent to the airline schedule planning process shown in Figure 1-1. The schedule design step is equivalent to frequency optimization step and the other three steps in Figure 1-1 together make up the vehicle optimization step.

The main objective in the frequency planning step is to maximize market share by offering services that match passengers' desired travel times subject to approximate considerations of available vehicles and crew, and maintenance restrictions. Teodorovic and Krcmar-Nozic [38] present a methodology that determines optimal flight frequencies on a network to maximize total profit and market share, and minimize the total schedule delay of all passengers on the network. They incorporate the approximate vehicle considerations by setting a maximum number of services in each market and a maximum number of seat-hours (a measure of airline's production level) for the entire system.

In the vehicle optimization step, we try to maximize vehicle utilization (or minimize vehicle costs), minimize crew costs, and satisfy maintenance constraints given the frequencies obtained in the first step. The complexities of these steps vary from one mode of transportation to another. For air transportation, the vehicle optimization step can be accomplished using the approaches of the last three boxes in Figure 1-1. (These steps are reviewed in the next section.)

For some modes of transportation (e.g., transit), the initial frequency plan can be slightly altered in order to optimize vehicle utilization [15]. Devanney [15] iterates frequency and vehicle planning until no further reduction in fleet size can be achieved. His heuristic appears to work well when demand is relatively stable, i.e., not very sensitive to schedule changes. However, this cannot easily be done in air transportation scheduling. Teodorovic and Krcmar- Nozic [38] find that demand for air transportation is very sensitive to frequency of service due to competitive conditions.

Other complexities particular to the scheduling process for air transportation include (Etschmaier and Mathaisel [17]):

1. The number of aircraft and frequencies per route are too small to permit rounding and the number of possible combinations are too large;

2. An aircraft can serve several markets: decomposing the network into smaller subsystems (e.g., markets) is impossible;

3. The number of feasible alternative ways of utilizing the fleet is high;

4. The crew costs are high and crew duty constraints are complex: thus crew scheduling considerations are extremely important;

5. A multitude of restrictions are imposed on operations by governmental regulations; and

6. Extensive service facilities are required at stations.

Schedule Evaluation

The purpose of *schedule evaluation* is to evaluate the profitability of the schedule. It takes into account the entire system including all airlines' flight schedules. Revenues are estimated from the fares collected from passengers carried. The number of requests in each itinerary is computed based on its *Quality Service Index* (QSI), a measure of relative attractiveness, which is a function of number of stops, level of service, time of departure, etc. The actual number of passengers carried is a function of the number of requests and aircraft capacity. Costs are estimated using flight operating costs, crew costs, maintenance costs and passenger related costs (such as baggage handling, meals, etc.), etc.

Etschmaier and Mathaisel [18] review some previous work in this area. One recent model is developed by Marsten et al. [25]. Their model takes every flight in the Official Airline Guide (OAG) as input, and the output is the estimated unconstrained demand for each itinerary. The resulting demand for a given airline is then fed to the fleet assignment model to determine the number of passengers carried and operating costs. From revenues and costs, estimated profits are then computed.

1.3 The Airline Schedule Planning Process

In this section, we review airline schedule design research, the central topic of this thesis, in detail. Then, we briefly review each of the other three steps in the airline schedule planning process.

1.3.1 The Schedule Design Problem

In general, the airline schedule design problem can be decomposed into 2 steps: (1) *route generation*—deciding which candidate flights should be considered; and (2) *route selection*—selecting the most profitable flight legs (or routes) out of all of the candidate flights. In this thesis, we focus only on the route selection step assuming that the route generation process has already been done and the list of candidate flights are given.

In the past, there have been efforts to improve the profitability of the schedule. Early literature includes Chan [10] and Simpson [31], for example. Chan [10] provides a framework for designing airline flight schedules covering both candidate route generation and route selection. However, he assumes that the demand in the markets is saturated, i.e., no appreciable demand can be stimulated when flights are added; thus, the demand and supply interaction is omitted. Simpson [31] presents a computerized schedule construction system that begins by generating demand using a gravity model, then solves the frequency planning problem. Finally, he constructs a flight schedule and solves the vehicle optimization problem upon that schedule. He does not address the problem of demand variations when the schedule changes.

It is very important to remark that all of the research cited above was performed before deregulation of the passenger air transportation industry in 1978. There are at least two obvious effects of this on the schedule design process. First, in candidate route generation, the number of eligible routes is significantly less due to regulations imposed by the United States Civil Aeronautics Board (CAB). Second, CAB regulation stabilizes the markets because schedule changes must be approved by the board, thereby delaying the implementation. Thus, the demand estimated for a market remains valid over a period of time in which there is no schedule change in the market.

For a more complete review of literature before 1985, interested readers are referred to Etschmaier and Mathaisel [17]. However, Dobson and Lederer [16] remark that most of the literature reviewed in Etschmaier and Mathaisel [17] addresses the problem of minimizing the airline's cost subject to *fixed transportation demand.*

More recent works in airline scheduling include for example Soumis, Ferland, and Rousseau [35], Dobson and Lederer [16], Nikulainen and Oy [26], Marsten et al. [25] and Berge [8].

Soumis, Ferland, and Rousseau [35] consider the problem of selecting passengers that will fly on their desired itinerary with the objective of minimizing spill costs. No recaptures are considered. Flight schedules are optimized by adding and dropping flights. When flights are added or dropped, their heuristic recalculates demands only in markets with significant amounts of traffic. Then, the passenger selection problem is resolved. Their method can be viewed as an enumeration of all possible combinations of flight additions and deletions.

Dobson and Lederer [16] first consider the problem of finding optimal fares that maximize schedule profit, given a fixed schedule. They develop a demand forecasting model considering fares, departure times, and travel times. They consider only one class of service, one size of aircraft, and they do not allow traffic to originate from or be destined to hub airports (i.e., only through traffic via hubs are considered). They use a two-stage heuristic to select an optimal set of flights between spoke and hub airports. Their heuristic starts with all candidate flights (placed at two-hour interval) in the schedule and eliminates non-profitable ones. It considers the candidate flights in order of their contributions from the highest to the lowest. The contributions are measured as the difference in profit (obtained from their profit maximizing model) between the schedule with every candidate flight included and the one with the considered flight dropped. In the second stage, after selecting a subset of flights, schedule feasibility is checked by solving fleet assignment problem.

Nikulainen and Oy [26] demonstrate the sensitivity of the total number of requests to the frequency in a market at different times of the day. They employ an exponential attractiveness function that captures the passengers' preferences when flights are offered at times other than their desired travel times. Their method can be used to find the optimal time of day for *an additional flight* such that the total number of passengers in the market are maximized. Network and vehicle considerations are not considered.

Marsten et al. [25] present a framework for incremental schedule design. Their approach is an enumeration of potential combinations of flight additions and deletions. Given a schedule, demands are estimated using the schedule evaluation model described earlier. Then, the fleet assignment problem is solved on the given schedule with the corresponding demands. The revenues are computed based on the passengers carried and costs are computed based on flight operating costs of the fleeted schedule. The profits from several sets of proposed additions and deletions are then compared to identify the highest one.

Berge [8] considers the problem of solving a subset of (given) candidate flights to augment the existing schedule such that *market coverage*, the probability that a random passenger finds at least one itinerary in his/her decision window, is maximized subject to available number of aircraft. The probability of any combination of flights is computed by numerical integration. Since the objective involves only market coverage, which is defined on *a random customer* and is well defined for each combination of flights, there is no notion of demand and supply interaction. His model is solved for a network containing 24 aircraft. He develops two solution approaches, one heuristic and the other an integer linear program. The heuristic approach tends to solve the problem quickly while maintaining an acceptable accuracy but relies on several assumptions. The integer linear programming approach has great flexibility but incurs long runtimes.

To recap, we can see that there have been efforts from the optimization community to address the airline schedule design problem. In chapter 3, we show how we address some of the shortcomings found in this literature.

1.3.2 The Fleet Assignment Problem

Typically, the fleet assignment model takes as input the available types and numbers of aircraft and a given schedule with fixed departure times. The costs of assigning an aircraft of type k to a flight leg i is the summation of the direct operating costs of flight i with an aircraft of type k and the *spill costs*, that is, costs incurred when insufficient capacity is assigned to flight legs and passengers are not accommodated, *are spilled*, on these flight legs. The objective of the fleet assignment problem is to find the minimum cost assignment of aircraft types to flight legs such that each flight leg is assigned to exactly one aircraft type; the number of flights assigned to an aircraft type into and out of a location are equal (or balanced); and the number of aircraft of each type assigned to the network does not exceed the number of each type available. Additional constraints considering maintenance requirements, noise and gate constraints can also be included.

The application of linear programming to fleet assignment problems can be traced back to as early as 1954 by Dantzig and Ferguson [13]. They consider the fleet assignment problem for non-stop routes. They formulate the problem as a linear program as opposed to a mixed integer program; thus, fractional solutions are allowed. However, fractional solutions might not be critical if the assignment is considered over some period of time.

Developments in this area through out the years has been impressive. Recent developments include Daskin and Panayotopoulos [14], Abara [1], Hane et al. [21], and Rexing [29]. Daskin and Panayotopoulos [14] present an integer programming model that assigns aircraft to routes and uses Lagrangian relaxation to obtain lower bounds on the optimal objective value and to develop heuristics to obtain a feasible solution. Abara [1] presents a model that uses the underlying connection arcs as decision variables which can lead to an explosion in the number of variables. A limitation of his model is that it does not allow different *turn times* (minimum ground service times) for different fleet types at various locations. The model upon which we base our work is presented by Hane et al. [21]. We review their model in detail in Chapter 2. Rexing [29] presents an expanded fleet assignment model that allows slight flight re-timing within small time window. We also review this model in greater detail in Chapter 2.

1.3.3 The Aircraft Routing Problem

The aircraft routing problem (also known as the aircraft maintenance routing problem) takes a fleeted schedule and the available number of aircraft for each fleet as input. In traditional fleet assignment process, maintenance requirements are modeled only approximately by ensuring a sufficient number of *maintenance opportunities* for each fleet type. A maintenance opportunity exists when an aircraft overnights at one of its maintenance locations. While this ensures that, on average, enough aircraft of each type are in maintenance nightly, it does not guarantee that individual aircraft are treated equally: one aircraft might have one maintenance opportunity per day while another might not have any in a week. The aircraft maintenance routing problem addresses this. It determines the actual *rotation*, or sequence of connected flights beginning and ending at the same location, of individual aircraft subject to maintenance rules imposed by both the regulatory agency and the airline itself. Often times the airline's rules are more stringent than these of the regulatory agency, to avoid the expensive penalty associated with violating maintenance rules.

Simpson [32] reviews several models for the aircraft routing problem. Recent works in this area include the works by Gopalan and Talluri [20], Clarke et al. [11], and Barnhart et al. [3].

1.3.4 The Crew Scheduling Problem

In crew scheduling problem, the objective is to find the minimum cost assignment of flight crews (pilots and flight attendants) to flight legs subject to several restrictions. For example:

1. pilots are qualified to fly only certain aircraft types;

2. work schedules must satisfy maximum *time-away-from- base* (the period that flight crews are away from their domicile stations) restrictions;

3. crews are not allowed to stay on duty longer than a *maximum flying time* requirement;

4. work schedules must satisfy *minimum rest time*; etc.

Crew scheduling problems are typically broken into two steps [7]:

1. *the crew pairing problem*, and

2. *the crew assignment problem.*

The objective of crew pairing problem is to find a set of work schedules that cover each flight the appropriate number of times and minimize total crew costs. In crew assignment, these pairings are combined with rest periods, vacations, training time, etc. to create extended work schedules that can be performed by an individual. The objective of the crew assignment problem is to find a minimum cost assignment of employees to these work schedules. There are two traditional approaches for crew assignment:

1. *rostering*, and

2. *bidline generation.*

With rostering, a common practice in Europe, schedules are constructed for specific individuals. A subset of schedules is selected such that total crew costs are minimized, each individual is assigned to a schedule, and all pairings in the crew pairing problem solution are contained in the appropriate number of schedules. With bidline generation, a common practice in North America, the cost-minimizing subset of schedules is selected without referring to specific individuals. Employees then reveal their relative preferences for these schedules through a *bidding* process. The airline assigns schedules to employees based on individual priority rankings, which are often related to seniority.

Crew pairing problems are usually formulated as *Set Partitioning* problems where each row corresponds to a scheduled flight and each column corresponds to a legal *crew paring* [24]. A pairing is composed of *duties*, separated by rest periods. A duty is a sequence of flight legs to be flown consecutively in one day that satisfies all work rules. In some instances, *deadheading* (flight crews flying on a flight as passengers for reposition) is allowed. Deadheading can be advantageous especially in long-haul crew pairing problems as shown by Barnhart et al. [4].

Vance et al. [39] present a formulation for crew pairing optimization with decision variables based on duty periods rather than pairings. Their formulation is able to improve the linear programming relaxation's lower bound on the optimal solution value, however, the formulation is more difficult to solve than traditional pairing based formulations.

1.4 Research Objective

Generally, flight schedules are not constructed entirely from scratch every time, rather they evolve from gradual adjustments made to previous schedules based on demand data that changes from season to season and many other considerations, including marketing strategies of the airlines and fleet composition and size. Constructing the entire schedule from scratch is a very ambitious and challenging task, both in theoretical and practical terms. The combinations of problem complexity and theoretical and computational limitations led us to embark on a less challenging task; that of *incremental schedule design.*

Incremental schedule design begins with a flight schedule and alters it to improve schedule profitability. Our methodology for improving the current schedule allows two major types of changes, namely:

1. It allows re-timing of flights in the existing schedule; and

2. It allows one or more flights to be added to or deleted from the current schedule.

For flight re-timings, we assume that the re-timings occur within small time windows, and so, the demand for that flight is not affected. When adding and/or deleting flights, the constant demand assumption no longer holds because these schedule changes might lead to significant changes in quality of service attributes, and hence, demand can be affected.

We present an *Incremental Schedule Design Model* (ISD) in chapter 3 that integrates flight re-timings, and addition and deletion decisions in order to improve upon a given schedule. Our ISD model is a combination of four existing models (each of which is reviewed in chapter 2), namely,

1. The fleet assignment model (FAM) [21],

2. The passenger mix model (PMM) [22],

3. The origin-destination fleet assignment model (ODFAM) [22], and

4. The fleet assignment with time windows model (FAMTW) [29].

The ISD model incrementally improves the profitability of the existing schedule by evaluating the effects of adding and removing given candidate flight legs and simultaneously exploring the opportunities for re-timings. (Note that we treat the candidate flight legs as given because we do not consider route generation in our process.) In ISD, we capture *approximately* the changes in unconstrained demand resulting from changes to the flight network.

Even this incremental approach to schedule design can result in prohibitively large and difficult to solve models. Consequently, we illustrate the potential value of modeling these various schedule changes by considering them separately. First we focus on the effect of additions and deletions of flight legs, keeping departure times fixed. Then, we focus on the effect of flight re-timings for a fixed set of flight legs. Although the combined value of both types of change exceeds the sum of the individual changes, we are able to estimate conservatively their combined impact by considering them separately.

Note that our ISD model integrates part of the schedule design problem with the fleet assignment problem. This integration represents another step towards the ultimate goal of developing tools that simultaneously (as opposed to sequentially) solve the airline schedule planning problem. There have been other such integrative efforts in Barnhart et al. [3], and Subramanian and Marsten [36]. In Barnhart et al. [3] the fleet assignment and aircraft routing problems are combined, and in Subramanian and Marsten [36] the fleet assignment and crew pairing problems are combined.

The ISD model can be used as a *fleet planning tool* also. Fleet planning involves determining the optimal fleet composition and size for future operations. Such planning is essential because aircraft ordering usually takes years to complete (starting from order placement until delivery of aircraft).

1.5 Contributions

The contributions of this thesis are summarized as follows:

1. We develop an incremental schedule design approach that

    - generates improved flight schedules by integrating schedule design and fleet assignment;

- constructs a framework and formulation for incremental design that considers addition, deletion and re-timing of flight legs simultaneously; and
- captures demand and supply interactions approximately.

2. We demonstrate our incremental schedule design methodology using data from a major U.S. airline:

    - In case study I, we consider additions and deletions of flight legs on a sample network with fixed departure times.
    - In case study II, we consider departure re-timings on a fixed flight network. We show the effects of re-timings under different scenarios, including a case investigating flow management in the U.S. national air space.

3. We show how our incremental schedule design model can also be used with a slight modification, to determine optimal fleet composition and size under different future operating scenarios.

1.6 Outline of the Thesis

In Chapter 2, we review the literature, model formulations, and solution approaches for four important schedule planning problems, namely, Fleet Assignment, Passenger Mix, Origin-Destination Fleet Assignment, and Fleet Assignment with Time Windows. In Chapter 3, we present the concept, formulation, example, and solution approach for our ISD model. Next, in Chapter 4, we present a case study examining the effects of flight leg additions and deletions. In Chapter 5, we present another case study examining the effects of flight re-timings. Our conclusions and future research directions are provided in Chapter 6. A glossary of technical words and phrases is provided in appendix A.

Chapter 2

Review of Four Airline Schedule Planning Models and Algorithms

2.1 Overview

In this chapter we review four important models that are essential to the development of the tools for incrementally designing improved flight schedules. The four models are:

1. the Fleet Assignment Model (FAM),
2. the Passenger Mix Model (PMM),
3. the Origin-Destination Fleet Assignment Model (ODFAM), and
4. the Fleet Assignment with Time Windows Model (FAMTW).

2.2 The Fleet Assignment Model

The fleet assignment model (FAM) that we refer to in this section is based on the work of Hane, et al. [21]. The objective of the fleet assignment model is to allocate aircraft types to flight legs based on a schedule that is fixed in solving the schedule design problem. In fleet assignment, the idea is to assign larger aircraft to the flights that have higher passenger demand, otherwise potential revenues are lost from *spilled passengers*, that is, passengers that are not accommodated and are lost to the airline. Similarly, smaller aircraft are placed on lower demand flights, because large aircraft have higher operating costs than small ones. Formally, the *basic fleet assignment* problem can be defined as follows (adapted from Rexing [29]):

> *Given a flight schedule with fixed departure times and costs (fleet-and-flight specific operating costs and spill costs), find the minimum cost assignment of aircraft types to flights, such that (1) each flight is covered exactly once by an aircraft, (2) flow of aircraft by type is conserved at each airport, and (3) only the available number of aircraft of each type are used.*

Typically, U.S. airlines consider a *daily* flight schedule, that is, one that repeats each day of the week. Mathematically, the fleet assignment problem can be formulated as shown in figure 2-1 (Hane, et al. [21]). $f_{k,i}$ is the binary variable that takes on value 1 when flight i is flown by fleet type k and 0 otherwise; $C_{k,i}$ is the cost of assigning fleet type k to flight i; y_{k,o,t^+} and y_{k,o,t^-} are the variables that count the number of aircraft of fleet type k at location o just after and just before time t respectively; y_{k,o,t_n} are the variables that count the number of aircraft for fleet type k, location o, at the count time t_n; $I(k, o, t)$ and $O(k, o, t)$ are sets of flights arriving and departing from location o at time t for fleet type k, respectively; O is the set of locations; $CL(k)$ is the subset of flight variables for fleet type k that are being flown at the count time; and N_k is the number of aircraft available for fleet type k. L and K are sets of flights and fleet types, respectively.

The objective function (Equation 2.1 of the fleet assignment model describes the cost of assigning the aircraft types to flight legs. This cost incorporates the operating costs and the spill costs. The constraints 2.2 are *cover constraints* ensuring that each flight is covered once and only once by a fleet type. Constraints 2.3 are *conservation of flow constraints* ensuring *aircraft balance* , that is, conservation of flow of aircraft by type throughout the network. Constraints 2.4 are *count constraints* ensuring that only the available number of aircraft of each type are used in the assignment.

The model shown here is the kernel of models implemented in the airline industry. Other considerations, not captured in this model, include noise control at locations, gate restrictions, exclusions of certain aircraft types at certain locations, etc. Their approach can solve the U.S. domestic fleet assignment problem with approximately 2,000 flights in 40 minutes on average on workstation class computer.

$$Min \sum_{i \in L} \sum_{k \in K} C_{k,i} f_{k,i} \tag{2.1}$$

Subject to:

$$\sum_{k \in K} f_{k,i} = 1 \quad \forall i \in L \tag{2.2}$$

$$y_{k,o,t^-} + \sum_{i \in I(k,o,t)} f_{k,i} - y_{k,o,t^+} - \sum_{i \in O(k,o,t)} f_{k,i} = 0 \quad \forall k, o, t \tag{2.3}$$

$$\sum_{o \in O} y_{k,o,t_n} + \sum_{i \in CL(k)} f_{k,i} \leq N_k \quad \forall k \in K \tag{2.4}$$

$$f_{k,i} \in \{0, 1\} \tag{2.5}$$

$$y_{k,o,t} \geq 0 \tag{2.6}$$

Figure 2-1: The Fleet Assignment Model (FAM)

Hane et al. [21] demonstrate solution techniques for this model using an airline network with 2600 flights and 11 fleet types. The techniques they employ include:

1. *node consolidation*: an algebraic substitution technique that lead to significant reductions in problem size;

2. *island construction*: an exploitation of special problem structure that achieves further reduction in problem size; and

3. *specialized branching strategies and priorities*: branching based on a special ordered sets and selection of variables on which to branch based on a measure of variability of the objective coefficients.

Their approach solves U.S. daily fleet assignment problems in approximately 30 minutes on workstation class computers.

2.3 The Passenger Mix Model

The passenger mix model we present was developed by Kniker [22]. It takes a *fleeted schedule* (that is, each flight leg is assigned one fleet type), and *unconstrained itinerary demand* as input and finds the most profitable flow of passengers over this schedule. Since the schedule is fleeted, the capacity (i.e., the number of passengers that can fly) on each leg is known. The objective of the model is to identify the best mix of passengers from each itinerary on

Table 2.1: PMM Model Example: Flight Schedule

Flight	Origin	Destination	Capacity
A	BOS	ORD	120
B	ORD	DEN	120

Table 2.2: PMM Model Example: Itinerary Level Demand

Itinerary	Origin	Destination	Demand	Fare
A	BOS	ORD	75	150
B	ORD	DEN	80	150
C	BOS	DEN	50	250

each flight leg. The model will deliberately spill passengers on less profitable itineraries in order to secure the seats for the passengers on more profitable itineraries. It is important to note that the profitability of an itinerary cannot be computed by evaluating only its legs. Rather, since several itineraries can share on flight leg, one itinerary can effect capacity on several flight legs, even those that are not contained in it.

To illustrate these network interactions, we consider the example of tables 2.1 and 2.2. Using a greedy algorithm (booking higher fare passengers first) 75 BOS- ORD passengers would be booked on flight A and 80 ORD-DEN passengers would be booked on flight B, and 40 seats would be booked on *both* flights for BOS-DEN passengers. This algorithm yields a revenue of $33,250. Alternatively, we could book all BOS-DEN passengers first and then assign the remaining seats on both flights to BOS-ORD and ORD-DEN passengers. This approach yields an increased revenue of $33,500. The maximum revenue, however, is $33,750 with 75 BOS-ORD passengers, 75 ORD- DEN passengers, and 45 BOS-DEN passengers assigned. From this tiny example, we can see that the mix of passengers on flights can affect revenues substantially.

The passenger mix problem can formally be defined as follows:

Given a fleeted flight schedule and the unconstrained itinerary- based demands, find the most profitable flow of passengers over the network, such that (1) the total passengers on each flight does not exceed the capacity of the flight, and (2) the total passengers on each itinerary does not exceed the unconstrained demand of that itinerary.

$$Max \sum_{p \in P} \sum_{r \in P} fare_r x_p^r \tag{2.7}$$

Subject to:

$$\sum_{p \in P} \sum_{r \in P} \delta_i^r \cdot x_p^r \quad \leq CAP_i \quad \forall i \in L \tag{2.8}$$

$$\sum_{r \in P} x_p^r / b_p^r \quad \leq D_p \quad \forall p \in P \tag{2.9}$$

$$x_p^r \quad \geq 0 \tag{2.10}$$

Figure 2-2: The Passenger Mix Model (PMM) Formulation.

The formulation of the passenger mix problem is shown in figure 2-2. The variables are defined as follow: x_p^r is the number of passengers who fly on itinerary r that desired travel on itinerary p, $fare_p$ is the average fare of itinerary p, CAP_i is the capacity of flight i, b_p^r is the *recovery rate* of passengers desiring itinerary p who are offered itinerary r (the recovery rate is defined as the fraction of passengers who accept the redirected itinerary offer), δ_i^r equals 1 if flight i covers itinerary p and 0 otherwise. L and P are sets of flights and itineraries, respectively.

The objective function of the passenger mix model (Equation 2.7) is to maximize revenues from the flow of passengers on itineraries. Constraints 2.8 are the capacity constraints that ensure that the number of passengers on a flight does not exceed the number of seats provided by the aircraft that is assigned to that flight. Constraints 2.9 are the demand constraints ensuring that the total number of passengers that is accommodated or spilled does not exceed the corresponding unconstrained itinerary-based demand.

The passenger mix model is applicable to the airline recovery problem (Kniker [22]). The problem addresses the question of how to recover the flight schedule when operations are disrupted by severe weather, mechanical failure, or delayed crews, for example. The PMM model can be used to re-route the affected passengers. The PMM model is also applicable to *revenue management.* The idea of revenue management is to maximize profits by booking low fare passengers when there is no high fare passenger demand. Kniker [22] shows that the PMM model can be used to compute an upper bound on the expected contribution that can be achieved by any revenue management process.

In PMM model, the number of variables is the number of itineraries squared. Hence, *column generation* techniques are necessary to solve the problems of the size faced by large U.S. airlines. Column generation methods start by solving a *restricted master problem* in which only some of the variables are included. The solution to the restricted master problem provides information about dual prices which will be used in solving the *pricing subproblem.* The pricing problem is to identify variables that might improve the current solution. These variables are added to the restricted master problem to create an augmented restricted master problem in the next iteration. The algorithm terminates when the pricing problems show that there are no variables that can improve the current solution. A summary of column generation techniques can be found in Ahuja et al. [2].

One disadvantage of the formulation given by figure 2-2 is the number of constraints 2.9. Even for a small network, the number of itineraries can be large. For example, in our case study with 228 flights, the number of itineraries is 608. Another schedule with approximately 2,000 flights has 76,500 itineraries. To overcome this, Kniker [22] proposed a new formulation, based on the work of Barnhart et al. [6], employing a change of variable strategy using *keypaths.* For the PMM model, the commodity is the passengers desiring travel on itinerary p and the keypath is the set of flights legs in the itinerary p. If many of the flight legs are not capacitated, then most passengers fly on their desired itinerary (the keypath) and we consider explicitly only those passengers that fly on alternative itineraries because of capacity restrictions. The proposed change of variable relationship is:

$$x_p^p = D_p - \sum_{r \neq P} t_p^r, \tag{2.11}$$

$$x_p^r = b_p^r t_p^r, \tag{2.12}$$

where t_p^r is defined as the number of passengers who desire travel on itinerary p, but the airline attempts to redirect onto itinerary r.

The new PMM model, using the change of variable, called the keypath PMM, is in figure 2-3 [22]. Constraints 2.15 are the capacity constraints. The term $\sum_{r \in P} \sum_{p \in P} \delta_i^p t_p^r - \sum_{p \in P} \delta_i^p t_p^p$ can be viewed as the number of passengers who are spilled from their desired itinerary p. The term $\sum_{r \in P} \sum_{p \in P} \delta_i^p b_r^p t_r^p - \sum_{p \in P} \delta_i^p b_r^p t_r^p$ is the number of passengers who are recaptured by the airline. (Note that we assume $b_p^p = 1$.) CAP_i is the capacity of the aircraft assigned to leg i, and Q_i is the unconstrained demand on flight leg i, which can be

$$Min \sum_{p \in P} \sum_{r \in P} (fare_p - b_p^r fare_r) t_p^r \tag{2.14}$$

Subject to:

$$\sum_{r \in P} \sum_{p \in P} \delta_i^p t_p^r - \sum_{r \in P} \sum_{p \in P} \delta_i^p b_r^p t_r^p \quad \geq Q_i - CAP_i \quad \forall i \in L \tag{2.15}$$

$$\sum_{r \in P} t_p^r \quad \leq D_p \quad \forall p \in P \tag{2.16}$$

$$t_p^r \quad \geq 0 \tag{2.17}$$

Figure 2-3: The Key-Path PMM Formulation.

written mathematically as,

$$Q_i = \sum_{p \in P} \delta_i^p D_p. \tag{2.13}$$

The last term of Equation 2.15 (CAP_i) is the capacity of flight i (recall that the schedule has been fleeted). Constraints 2.16 are the demand constraints.

Although the keypath formulation still requires the same number of constraints as formulation 2.7 - 2.10, the second set of constraints can be relaxed initially. To understand why, observe that most of the objective function coefficients of 2.7 are positive. Even though most of the time the fare of itinerary r is higher than that of itinerary p, the actual fare collected is scaled down by the factor b_p^r, the recapture rate, which can be a small number. Notice also that if r is the same as p, i.e., the passengers are not redirected to any other itineraries and the net effect on the objective function is zero. Since most of the objective function coefficients are positive, an optimal solution reduces t_r^p values as much as possible. Therefore, most of constraints2.16 will not be binding.

Row generation techniques are used to solve model 2.14 - 2.17 with constraints 2.16 eliminated initially. In general, row generation techniques neglect subsets of constraints in the *restricted master problem* and then after solving the restricted master problem, a *separation problem* is solved to identify violated constraints. These constraints are added to the restricted master problem and the process is repeated until the restricted master problem solution satisfies *all* constraints. A summary of row generation is in Bertsimas and Tsitsiklis [9]. Kniker et al. [22] have shown that for the keypath PMM model, less than 6% of constraints 2.16 must be added explicitly to the model, when solving problems with approximately 2000 flights, 9 fleet types, and 76,500 itineraries.

$$Min \sum_{i \in L} \sum_{k \in K} C_{k,i} f_{k,i} + \sum_{p \in P} \sum_{r \in P} (fare_p - b_p^r fare_r) t_p^r \tag{2.18}$$

Subject to:

$$\sum_{k \in K} f_{k,i} = 1 \quad \forall i \in L \tag{2.19}$$

$$y_{k,o,t^-} + \sum_{i \in I(k,o,t)} f_{k,i} - y_{k,o,t^+} - \sum_{i \in O(k,o,t)} f_{k,i} = 0 \quad \forall k, o, t \tag{2.20}$$

$$\sum_{o \in O} y_{k,o,t_n} + \sum_{i \in CL(k)} f_{k,i} \leq N_k \quad \forall k \in K \tag{2.21}$$

$$\sum_{k \in K} CAP^k f_{k,i} + \sum_{r \in P} \sum_{p \in P} \delta_i^p t_p^r - \sum_{r \in P} \sum_{p \in P} \delta_i^p b_r^p t_r^p \geq Q_i \quad \forall i \in L \tag{2.22}$$

$$\sum_{r \in P} t_p^r \leq D_p \quad \forall p \in P \tag{2.23}$$

$$f_{k,i} \in \{0, 1\} \tag{2.24}$$

$$y_{k,o,t} \geq 0 \tag{2.25}$$

$$t_p^r \geq 0 \tag{2.26}$$

Figure 2-4: The Origin-Destination Fleet Assignment Model (ODFAM) Formulation.

2.4 The Origin-Destination Fleet Assignment Model

The *Origin-Destination Fleet Assignment Model (ODFAM)* integrates two basic models: the Fleet Assignment Model (FAM) and the Keypath Passenger Mix Model (PMM) (Kniker et al. [22]). The motivation for the origin-destination fleet assignment model is that FAM does not accurately capture revenue because it ignores interactions between flight leg and demands. As stated earlier, FAM requires as input the revenue for each fleet-type- flight-leg combination (or equivalently the spill cost for each fleet-type-flight-leg combination). This is impossible to compute exactly, however, until a fleeting is known. In ODFAM, the dependences of these decisions on one another are modeled and operating costs and revenues can be computed much more accurately since the fleet assignment and passenger mix problems are solved simultaneously.

To solve these two problems simultaneously, we integrate the two models into one and remove the revenue (or spill) from the objective coefficients of the fleet assignment model because it is accounted for by the objective coefficients of the passenger mix model. The objective of the combined model is then to minimize the total operating costs less the total revenue; or equivalently, to minimize the total operating costs and the spill costs.

Figure 2-4 details ODFAM. The variable definitions are the same as those in the FAM and keypath PMM models, and CAP_k denotes the capacity of fleet type k. The first three sets of constraint (Equations 2.19 to 2.21) are FAM's constraints and the next two constraints (Equations 2.22 and 2.23) are the PMM's constraints. Note however, that there is a change in one of the terms in the capacity constraints 2.22. That is, the first term on the left has been moved from the right-hand-side because now, the capacity of the flight is also a variable.

The size of ODFAM can become prohibitively large. All of the techniques mentioned previously, namely, node consolidation, island construction, and column and row generation must be utilized in order to solve the problem. In larger instances, even with these techniques, Kinker [22] reports solutions sometimes cannot be obtained.

The ODFAM algorithm used by Kinker [22] initially includes only fleet-type-flight-leg-combination variables ($f_{k,i}$), ground variables ($y_{k,o,t}$), and a subset of traffic variables (t_p^r) in the master problem. Initially, all constraints except 2.23 are included in the restricted master problem. His algorithm iteratively solves the restricted master problem and iteratively generates columns and rows until the LP relaxation of ODFAM is solved. He embeds this in a branch and bound scheme to ensure integrality of the fleet assignment variables.

2.5 The Fleet Assignment with Time Windows Model

The motivation for the development of the fleet assignment with time windows model (FAMTW) (Rexing et al. [29]) is that by providing time windows within which flights can depart, a more cost effective fleeting and schedule might be obtained. That is , the output of FAMTW is both a fleet assignment and selected departure times (within a pre-specified time windows) for each flight leg. FAMTW can lead to reductions in fleet assignment costs in two ways:

1. opportunities arise to assign a more appropriate aircraft type to a flight leg when flight departure are re-timed because more aircraft connections are possible, and

2. aircraft can be more efficiently utilized given re-timings in the flight schedule and this can result in a fewer number of aircraft needed to fly the schedule.

Table 2.3: Flight Connection Example

Flight	Origin	Destination	Departure Time	Ready Time	Demand
A	BOS	ORD	0800	1000	150
B	ORD	DEN	0955	1200	150

To illustrate the idea of time windows, consider Table 2.3 (Rexing [29]). The demands on both flights A and B are the same. Therefore, it may be appropriate to assign the same aircraft to both flight legs. However, this is currently impossible because the ready time of flight A is later than the departure time of flight B. By either allowing flight A to depart a little earlier or flight B to depart a little later (or both), we can fly both flights with a single aircraft.

The model formulation is given in figure 2-5. The variable definitions are the same as in FAM except $f_{n,k,i}$ is the binary variable that takes on value 1 if copy n of flight i is covered by fleet type k and 0 otherwise and $N_(ki)$ is the set of copies of flight i for fleet type k. Flight copies of a flight represent that same flight at different departure times. The *copy interval* defines how far apart a copy is placed from its preceding copy. At the extreme, copies are placed every minute. The objective function (Equation 2.27 and the constraints 2.28 - 2.30 are modified from FAM by replacing $\sum_{k\in K} f_{k,i}$ by $\sum_{k\in K}\sum_{n\in N_{ki}} f_{n,k,i}$. The second summation arises because only one of the copies needs to be flown and the departure times of the selected copy represents the departure time for that flight.

To specify the number of copies for each flight, we need two parameters: (1) *time window width*—the allowable time window within which the departure can be shifted, and (2) *copy interval*—the time between two consecutive copies of the same flight.

Since FAMTW requires copies for each fleet-type-flight-leg- combination variable, the number of variables can grow rapidly if window width is wide and/or a small copy interval is selected. Rexing et al. [29] uses the preprocessing techniques suggested by Hane et al. [21] to improve the solvability of their FAMTW algorithms. Rexing et al. propose two solution algorithms, namely, a *direct solution technique* (DST) and an *iterative solution technique* (IST). The direct solution technique can be viewed as a brute-force approach, i.e., it loads the entire preprocessed problem into the solver. The iterative solution technique reduces memory size requirements by exploiting the fact that only selected flight copies are in an

$$Min \sum_{i \in L} \sum_{k \in K} \sum_{n \in N_{ki}} C_{n,k,i} f_{n,k,i} \tag{2.27}$$

Subject to:

$$\sum_{k \in K} \sum_{n \in N_{ki}} f_{n,k,i} \quad = 1 \quad \forall i \in L \tag{2.28}$$

$$y_{k,o,t^-} + \sum_{i \in I(k,o,t)} \sum_{n \in N_{ki}} f_{n,k,i}$$

$$-y_{k,o,t^+} - \sum_{i \in O(k,o,t)} \sum_{n \in N_{ki}} f_{n,k,i} \quad = 0 \quad \forall k, o, t \tag{2.29}$$

$$\sum_{o \in O} y_{k,o,t_n} + \sum_{i \in CL(k)} \sum_{n \in N_{ki}} f_{n,k,i} \quad \leq N_k \quad \forall k \in K \tag{2.30}$$

$$f_{n,k,i} \quad \in \{0,1\} \tag{2.31}$$

$$y_{k,o,t} \quad \geq 0 \tag{2.32}$$

Figure 2-5: The Fleet Assignment Model with Time Windows (FAMTW) Formulation.

optimal solution. The difficulty is to identify which flight copies are necessary. The IST algorithm addresses this difficulty by first solving the fleet assignment problem on a special network containing only *reduced-duration flight arcs*, that is, flight arcs that begin at the latest possible times and end at the earliest possible times. The resulting fleet assignment is used to partition the flights by assigned fleet type. Then, for each fleet, feasibility of the assignment is checked using original flight times and special arcs called *backward arcs* that identify infeasible assignments. The flights associated with these infeasible assignments are the flights whose reduced- duration flight arc are replaced by flight copies in the next iteration, the fleet assignment problem is solved over this hybrid network. The process repeats until no infeasible assignments exist and hence, an optimal assignment has been found.

Chapter 3

Incremental Schedule Design: Model Framework, Formulation, and Solution Algorithms

3.1 Model Framework Development

We begin by first describing the terminology of *open flights.* Open flights are candidate flights that are being considered as possible additions or deletions from the current schedule. The list of open flights contains two components:

1. flights that exist in the current schedule but are being proposed for deletion, and
2. flights that do not exist in the current schedule but are being proposed for addition.

Flights that currently exist in the schedule but are not being proposed for deletion are referred to as *fixed flights.*

As described earlier, there are two fundamental changes that we can make to the schedule, namely,

1. re-timing flights within small time windows, and
2. adding to and/or deleting from the schedule subsets of open flights.

Therefore, there are 4 alternative approaches to the problem:

1. allowing only flight re-timings,

2. allowing only flight additions and/or deletions,

3. allowing both flight re-timings and flight additions and/or deletions sequentially, and

4. allowing both flight re-timings and flight additions and/or deletions simultaneously.

The complexity of the problem becomes more involved when we move from the first to the last alternative. In the first alternative in which only flight re-timings within small time windows are allowed, the demands are constant; therefore, the demand and supply interactions can be neglected. In the second alternative, we need to explicitly address the demand and supply interactions because adding (removing) flights to (from) the schedule can have serious ramifications on demand distributions through out the network. In the third alternative, we simply apply the first two alternatives sequentially. Either a one-pass approach or an iterative approach can be adopted for the third alternative. The last alternative considers all potential incremental changes simultaneously. It is obvious that the solution from this last alternative cannot be worse than those from the other three alternatives.

3.1.1 Tools for Incremental Schedule Design

The FAMTW model developed by Rexing, et al. [29] can be applied directly to the incremental schedule design problem that considers only flight re-timings. There are no existing models that are readily applicable to the incremental schedule design problem that considers flight additions and deletions. We, therefore, develop the Incremental Schedule Design for Flight Additions and Deletions model or ISD-A/D model based on the FAM (Hane et al. [21]), PMM (Kniker et al. [22]), and ODFAM (Kniker et al. [22]) models. The ISD-A/D model explicitly addresses demand and supply interactions using *demand correction terms* that adjust unconstrained demands according to the status of the flights in the schedule. To adopt the third approach, one can simply solve FAMTW and ISD-A/D models sequentially. The last approach, on the other hand, requires the integration of FAMTW and ISD-A/D models. We referred to this combined model as the Incremental Schedule Design (ISD) model.

3.2 The Incremental Schedule Design Models

In this section, we show how we capture the changes in demand when flight status changes. Next we describe the ISD-A/D and ISD models. Then, we provide a solution algorithm for these models.

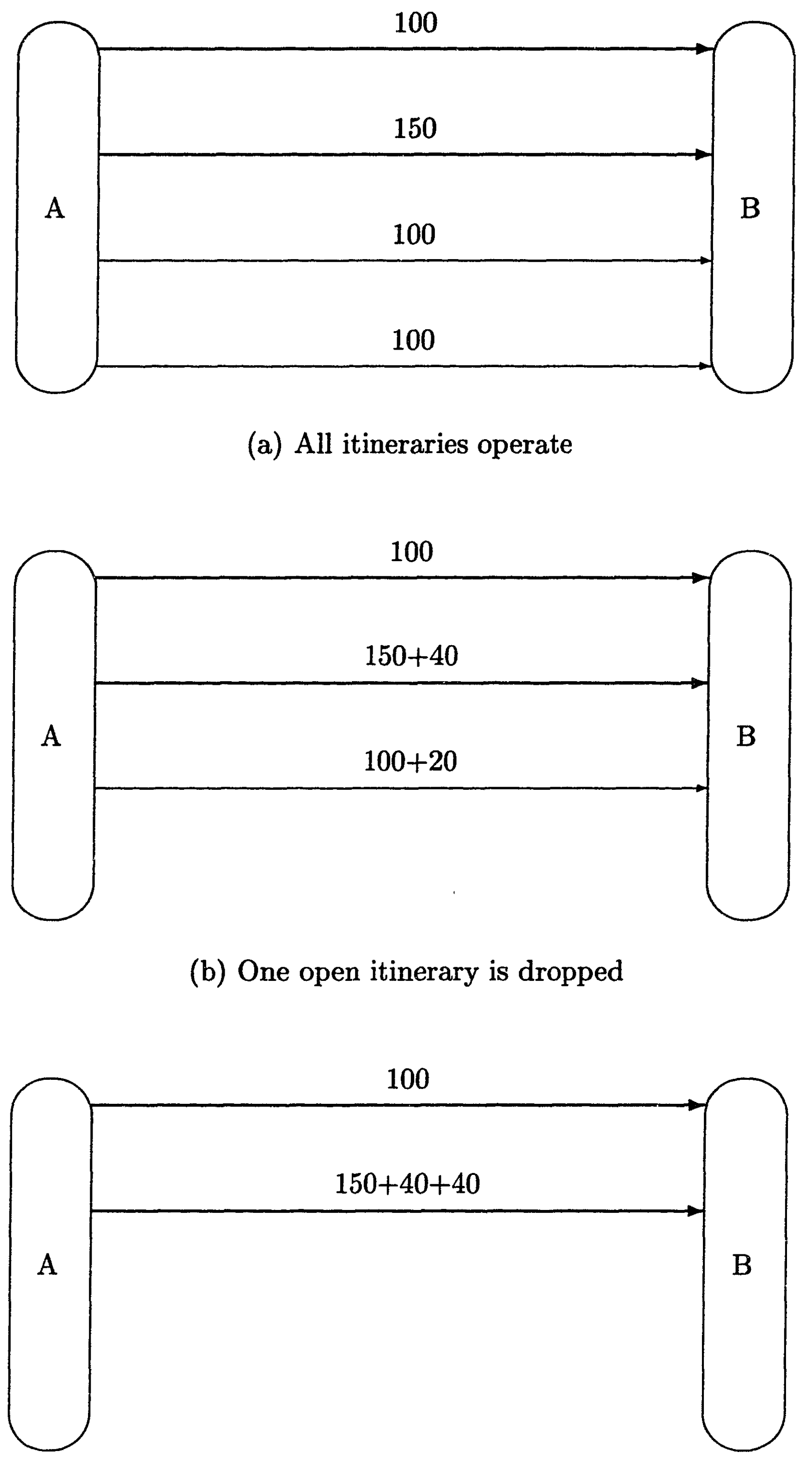
3.2.1 The Treatment of Demand and Supply Interactions

We assume that markets are isolated from one another. Specifically, we do not allow interactions among *parallel markets*, that is, any change in a market does not affect the demand in its parallel markets. For non-parallel markets, this assumption is automatically satisfied. Thus, the changes in demand are market specific and the changes are contained only in that market. This enables us to focus our attention at the market level and adjust the changes in demand for each market separately.

The best way to understand our approach for capturing changes in demand as schedules change is through an example. Figure 3-1 (a) depicts the situation of a market with 4 itineraries. The first and second itineraries are *fixed itineraries*, i.e., they contain only fixed flights. The third and forth itineraries are *open itineraries*, i.e., each of these itineraries contain at least one open flight. We further assume that the first itinerary is in the morning and the last three itineraries are in the afternoon.

The numbers on the arcs are the unconstrained demands (number of requests) when all itineraries exist in the schedule, that is, we assume that open itineraries are initially flown. Each itinerary has 100 requests except the second itinerary, which has 150 requests. (We can view the second itinerary as a nonstop itinerary and the rest as connecting itineraries.) Thus, there are a total of 450 requests when both open itineraries are flown.

In Figure 3-1 (b), we assume that one of the open flights in the last itinerary is deleted and, as a result, the last itinerary no longer exists. At first glance, it may appear that 100 potential customers are lost. This need not be true. Some of these 100 potential customers will go to other airlines, and some of them will return to our airline. Those that are coming back to our airline will request other itineraries that still exist in the schedule (itineraries 1-3 in this case). We assume that 40 and 20 of the 100 requests previously on the fourth itinerary will request the second and third itineraries, respectively. The first itinerary does not receive any additional requests because it is in the morning and the 100



(a) All itineraries operate

(b) One open itinerary is dropped

(c) Two open itineraries are dropped

Figure 3-1: Our Approach for Capturing Demand and Supply Interaction passengers presumably prefer the itinerary that departs close to the time (the afternoon) of their original itinerary. The second itinerary receives more requests than the third because the former is nonstop while the later is connecting. Thus, the total requests for our airline are now reduced to 410. We lost 40 requests to our competitors.

In Figure 3-1(c), we assume further that the third itinerary is also cancelled. The same phenomenon occurs. The second itinerary receives 40 more requests from the 100 requests cancelled from the third itinerary. The first itinerary again receives none. Now, the total request is reduced to 330. We lost 120 requests in total. The lost requests when two itineraries are deleted more than double those when only one itinerary is deleted.

This shows that there is a non-linear relationship between market share and service frequency. We refer to the adjustments in demand resulting from changes in the flight schedule as *demand correction terms*. They correct the unconstrained demands for other itineraries when open itineraries are dropped. Note that the corrections become approximate when there is more than one open itinerary deleted. To see why, return back to Figure 3-1, assume that we know exactly that when an open itinerary is cancelled, 40 requests will go to the nonstop itinerary, and 20 requests will go to the connecting itinerary. This is true for both itineraries 3 and 4. In Figure 3-1(c), our approach predicts that the total request is 330, based on the correction terms for cancellation of *one itinerary at a time*. The total request of 330 becomes an approximation of the true number. The combined effect of canceling two itineraries might decrease the total request further or might not harm the total request so much. We explain how this problem can be fixed, in theory, after presenting the model formulation. We do not adopt such high levels of interaction, however, in order to keep our model tractable.

It is very important to remark that the validity of the resulting schedule crucially hinges on the accuracy of demand correction terms. Even though the changes become approximate when we consider more than one open itinerary, we need to have the first order correction for cancellation of one itinerary as accurate as possible. A sensitivity analysis of the model on these correction terms is necessary in order to identify the level of accuracy that is needed for the model.

3.2.2 The ISD-A/D Model

Figure 3-3 shows the formulation for the ISD-A/D model. For convenience, we list all of the variables and notations in Figure 3-2. In this formulation, our convention is that all unconstrained itinerary demands are initially considered for the schedule with all of the open flights being flown.

The objective function of our ISD-A/D model (Equation 3.1) is the same as that of ODFAM (Equation 2.18). It minimizes operating costs and spill costs. Constraints 3.2 are cover constraints for fixed flights ensuring that every fixed flight has to be assigned to a fleet type. Constraints 3.3 are cover constraints for open flights allowing the model to choose whether or not to fly flight i in the resulting schedule; if flight i is selected, a fleet type has to be assigned to it. Constraints 3.4 ensure the conservation of aircraft flow. Constraints 3.5 are count constraints ensuring that only available numbers of aircraft are used. Constraints 3.6 are capacity constraints ensuring that the number of passengers on flight i does not exceed its capacity. Constraints 3.7 are demand constraints ensuring that we do not spill more passengers than there are in the itinerary. Constraints 3.8 - 3.9 are *itinerary status constraints* that control the {0,1} variable, Z_q, for itinerary q. They ensure that Z_q takes on value 1 when itinerary q is flown in the schedule, and 0 otherwise.

It is easy to see that the ISD-A/D model is built on the FAM, PMM, and ODFAM models. Constraints 3.2 - 3.5 are modified FAM constraints. Constraints 3.6 - 3.7 are modified PMM and ODFAM constraints and constraints 3.8 - 3.9 are added in order to capture the demand and supply interactions.

Constraints 3.7 are similar to constraints 2.23 except that there is an additional term, $\sum_{q\in P^O} \Delta D_q^p(1-Z_q)$. This summation corrects the unconstrained demand for itinerary p as a result of canceling any itinerary $q \in P^O$. The additional term in constraints 3.6 $(\sum_{p\in P}\sum_{q\in P^O} \delta_i^p \Delta D_q^p(1-Z_q))$also serves the same purpose but at the flight level.

We described earlier that we could capture demand corrections more accurately when two or more itineraries are cancelled at the same time. This can be done by adding another set of {0,1} variable that indicates the status of *combinations* of itineraries and associating additional demand correction terms with these variables. For example, in Figure 3-1(c), we could add a second order correction term to capture correctly the change when the third and fourth itineraries are both cancelled. Specifically, the number of requests for

$f_{k,i}$	equals 1 if flight i is assigned to fleet k, 0 otherwise
$f_{n,k,i}$	equals 1 if copy n of flight i is assigned to fleet k, 0 otherwise
y_{k,o,t^-}	number of aircraft of fleet k, at location o, before time t
y_{k,o,t^+}	number of aircraft of fleet k, at location o, after time t
y_{k,o,t_n}	number of aircraft of fleet k, at location o, at the count time t_n
t_r^p	passengers (can be fractional) who want itinerary p but airlines attempt to redirect to itinerary r
Z_q	equals 1 if itinerary q is in the schedule, 0 otherwise
$C_{k,i}$	cost of assigning flight i to fleet k
$C_{n,k,i}$	cost of assigning copy n of flight i to fleet k
$fare_p$	fare of itinerary p
δ_i^p	equals 1 if itinerary p contains flight i, 0 otherwise
CAP^k	capacity of fleet k
b_r^p	recapture rate of itinerary p from itinerary r
ΔD_q^p	demand correction term for itinerary p as a result of canceling itinerary q
N_k	number of aircraft in fleet k
Q_i	unconstrained demand on flight i *when all itineraries are flown* (Equation 2.13)
D_p	unconstrained demand on itinerary p *when all open itineraries are flown*
N_q	number of flight in itinerary q
L^F	set of fixed flights
L^O	set of open flights
N_{ki}	set of $f_{n,k,i}$ (copies of $f_{k,i}$)
O	set of locations
K	set of fleets
$CL(k)$	set of (copies of) flights that cross count line
$\{k,o,t\}$	a node in the network specified by fleet k, location o, and time t
N	set of all nodes ({k,o,t})
$I(k,o,t)$	set of flights that fly into node {k,o,t}
$O(k,o,t)$	set of flights that fly out of node {k,o,t}
P	set of itineraries
P^O	set of open itineraries
$L(q)$	set of flights in itinerary q

Figure 3-2: Variable Definitions and Notations for ISD models

$$Min \sum_{i \in L} \sum_{k \in K} C_{k,i} f_{k,i} + \sum_{p \in P} \sum_{r \in P} (fare_p - b_p^r fare_r) t_p^r \tag{3.1}$$

Subject to:

$$\sum_{k \in K} f_{k,i} = 1 \qquad \forall i \in L^F \tag{3.2}$$

$$\sum_{k \in K} f_{k,i} \leq 1 \qquad \forall i \in L^O \tag{3.3}$$

$$y_{k,o,t^-} + \sum_{i \in I(k,o,t)} f_{k,i} - y_{k,o,t^+} - \sum_{i \in O(k,o,t)} f_{k,i} = 0 \qquad \forall \{k,o,t\} \in N \tag{3.4}$$

$$\sum_{o \in O} y_{k,o,t_n} + \sum_{i \in CL(k)} f_{k,i} \leq N_k \qquad \forall k \in K \tag{3.5}$$

$$\begin{aligned} \sum_{p \in P} \sum_{q \in P^O} \delta_i^p \Delta D_q^p (1 - Z_q) + \sum_{k \in K} CAP^k f_{k,i} & \\ + \sum_{r \in P} \sum_{p \in P} \delta_i^p t_p^r - \sum_{r \in P} \sum_{p \in P} \delta_i^p b_r^p t_r^p & \geq Q_i \qquad \forall i \in L \end{aligned} \tag{3.6}$$

$$\sum_{q \in P^O} \Delta D_q^p (1 - Z_q) + \sum_{r \in P} t_p^r \leq D_p \qquad \forall p \in P \tag{3.7}$$

$$Z_q - \sum_{k \in K} f_{k,i} \leq 0 \qquad \forall i \in L(q) \tag{3.8}$$

$$Z_q - \sum_{i \in L(q)} \sum_{k \in K} f_{k,i} \geq 1 - N_q \qquad \forall q \in P^O \tag{3.9}$$

$$f_{k,i} \in \{0,1\} \tag{3.10}$$

$$Z_q \in \{0,1\} \tag{3.11}$$

$$y_{k,o,t} \geq 0 \tag{3.12}$$

$$t_p^r \geq 0 \tag{3.13}$$

Figure 3-3: The ISD-A/D Model Formulation.

the second itinerary would become 150+40+40+x, where x is the second order correction term associated with cancellation of both the third and fourth itineraries. Note that x could take on either positive or negative value. Even though this is possible in theory, the model becomes intractable in practice. Further study (e.g., sensitivity analysis) is needed to indicate whether we need these higher-order correction terms.

3.2.3 The ISD Model Formulation

The ISD model can be constructed by combining the FAMTW and ISD- A/D models. The integration is straightforward. Figure 3-4 shows the ISD model formulation. The variable definitions and notations can be found in Figure 3-2. Note that we still assume that demand remains constant with flight re-timings.

Even though model integration is straightforward, model solution is not. Compared to FAM, the problem size of ISD grows far more quickly in terms of both number of variables and number of constraints. Currently, ODFAM is difficult to impossible to solve for realistic size networks (Kniker et al. [22]); thus, solving the ISD model on any realistic size problem is not possible at the moment. We present two case studies in Chapters 4 and 5 that separately address the re-timing decisions and addition/deletion decisions. The ISD model shown here serves as a framework for future development.

3.2.4 Solution Approach

The ISD model takes as input the list of fixed and open flights, demand data as well as correction terms, and fleet composition and size. Demands and correction terms are computed based on the *complete schedule*, that is, the schedule including all flights. If re-timings are allowed, time window widths and copy intervals need to be specified as well. The output of the ISD model includes:

1. a list of selected open flights to be flown,

2. an optimal fleeting for flights that are flown,

3. an optimal mix (specifying routing and numbers) of passengers, and

4. optimal departure times, if re-timings are considered.

$$Min \sum_{i\in L}\sum_{k\in K}\sum_{n\in N_{ki}} C_{n,k,i} f_{n,k,i} + \sum_{p\in P}\sum_{r\in P}(fare_p - b_p^r fare_r)t_p^r \tag{3.14}$$

Subject to:

$$\sum_{k\in K}\sum_{n\in N_{ki}} f_{n,k,i} = 1 \quad \forall i \in L^F \tag{3.15}$$

$$\sum_{k\in K}\sum_{n\in N_{ki}} f_{n,k,i} \leq 1 \quad \forall i \in L^O \tag{3.16}$$

$$y_{k,o,t^-} + \sum_{(i,n)\in I(k,o,t)} f_{n,k,i} - y_{k,o,t^+} - \sum_{(i,n)\in O(k,o,t)} f_{n,k,i} = 0 \quad \forall \{k,o,t\} \in N \tag{3.17}$$

$$\sum_{o\in O} y_{k,o,t_n} + \sum_{(i,n)\in CL(k)} f_{n,k,i} \leq N_k \quad \forall k \in K \tag{3.18}$$

$$\sum_{p\in P}\sum_{q\in F^O} \delta_i^p \Delta D_q^p (1 - Z_q) + \sum_{k\in K}\sum_{n\in N_{ki}} CAP^k f_{n,k,i} + \sum_{r\in P}\sum_{p\in P} \delta_i^p t_p^r - \sum_{r\in P}\sum_{p\in P} \delta_i^p b_r^p t_r^p \geq Q_i \quad \forall i \in L \tag{3.19}$$

$$\sum_{q\in P^O} \Delta D_q^p (1 - Z_q) + \sum_{r\in P} t_p^r \leq D_p \quad \forall p \in P \tag{3.20}$$

$$Z_q - \sum_{k\in K}\sum_{n\in N_{ki}} f_{n,k,i} \leq 0 \quad \forall i \in L(q) \tag{3.21}$$

$$Z_q - \sum_{i\in L(q)}\sum_{k\in K}\sum_{n\in N_{ki}} f_{n,k,i} \geq 1 - N_q \quad \forall q \in P^O \tag{3.22}$$

$$f_{n,k,i} \in \{0,1\} \tag{3.23}$$

$$Z_q \in \{0,1\} \tag{3.24}$$

$$y_{k,o,t} \geq 0 \tag{3.25}$$

$$t_p^r \geq 0 \tag{3.26}$$

Figure 3-4: The ISD Model Formulation.

Note that the mix of passengers we obtained from our ISD model is conditioned on the accuracy of the demand correction terms. We can obtain the (estimated) revenue for the resulting schedule from a separate calculation based on this mix of passengers.

In theory, a solution approach can be outlined as shown in Figure 3-5. The idea is first to obtain relatively accurate demand correction terms for as many combinations of flight additions and deletions as possible. Higher order correction terms can be included if necessary. Then, one run of the ISD model is run to determine the optimal subset of flights to be flown based on the correction terms that are initially prepared. In theory, if we estimated exactly all higher-order correction terms using our schedule evaluation model and included all of them in our ISD model, we could find an optimal schedule by solving ISD once. This strategy is impractical, however, because exponentially many runs of the schedule evaluation model (one run for each possible combination of flight additions and deletions) are necessary to estimate the correction terms exactly.

Consequently, we adopt the solution approach outlined in Figure 3-6. Instead of trying to obtain exact demand correction terms at the outset, we use rough estimates of these terms and try to revise them iteratively as we solve the problem. We add higher order correction terms as needed.

To start our approach, we obtain demand estimates for the full schedule and the corresponding set of demand correction terms using the schedule evaluation model. An initial estimate of the demand correction terms can be obtained, for example, by looking at cases when only one flight is added or deleted at a time, ignoring higher order terms.

In step 1 of our approach in Figure 3-6, we solve the ISD model to obtain a fleeted schedule. In step 2, a schedule evaluation model is called to determine the new set of demands based on the schedule resulting from step 1. Note that this is the demand for the schedule from step 1. In step 3, we solve the PMM model on the schedule from step 1 with demand data from step 2 to estimate revenue. The revenue from step 3 is then compared to that from step 1. If they are close to each other, we have captured the changes in demand quite well and the procedure is stopped; otherwise the demand correction terms are revised using the demand data from step 2. After revising demand correction terms, we resolve the ISD model in step 1 and follow the same procedure. The solution algorithm for ISD itself follows the one for ODFAM, outlined in section 2.4. We emphasize that at the beginning of every iteration an ISD problem is solved on the same full schedule but with revised demand

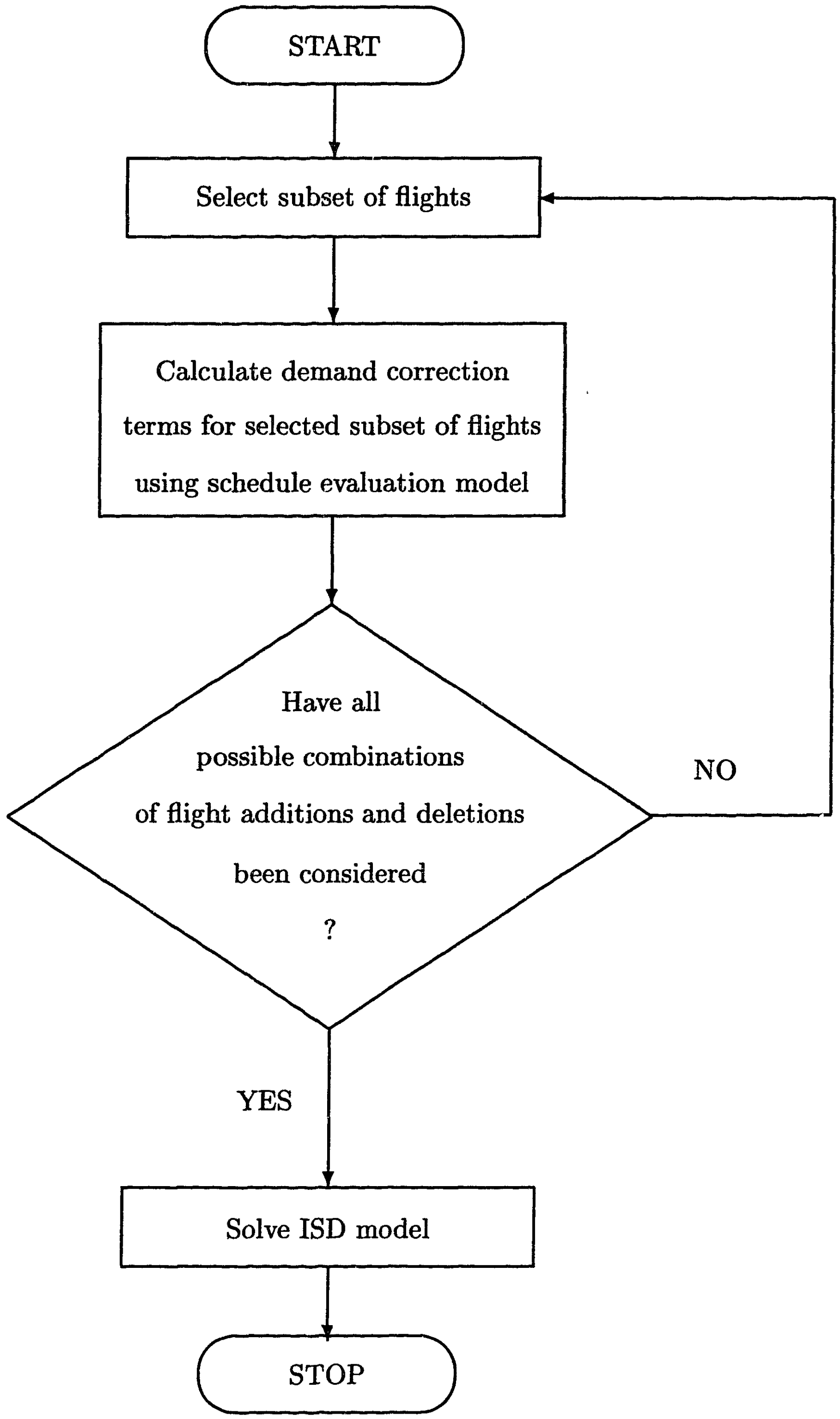


Figure 3-5: The Flowchart for Solution Approach I.

Table 3.1: Initial Demand Correction Terms

Dropped Itinerary	Demand Correction Terms on Itinerary			
	1	2	3	4
3	0	40	N/A	20
4	0	40	20	N/A

correction terms.

No rigorous statements can be made about algorithmic convergence. Convergence of the algorithm depends on the sensitivity of the model to these correction terms. If the model is very sensitive to these terms, it could take many iterations in order to get them right. Moreover, it could reveal that we might never get them "right" unless we use higher-order correction terms. If it is rather insensitive to these terms, we can use less accurate correction terms, and fewer iterations might be required.

Since every iteration revises the accuracy of demand correction terms, we might be able to capture changes in demands more accuracy as we iterate. Whenever there is only first degree correction (i.e., only one open itinerary is cancelled from a market), the correction terms associated with that itinerary in that market are revised to the exact values. However, whenever there are two or more cancelled itineraries, the correction terms might be approximate.

Example

To better understand the algorithm, consider the following example referring back to Figure 3-1. Recall that the third and fourth itineraries are open itineraries. The initial estimates of demand correction terms associated with open itineraries are summarized in Table 3.1. These initial values are fed into the ISD model in Step 1. Suppose that, in the first iteration, the resulting schedule from Step 1 does not include the fourth itinerary (Figure 3-1(b)). In Step 2, we assume that the schedule evaluation model outputs the unconstrained demands of 105, 188, and 115 for the first to the third itineraries, respectively. Since in this situation, there is only one itinerary being dropped, the demand correction terms can be revised to exact values. Table 3.2 summarizes the revised and exact correction terms for the fourth itinerary, and the not-yet-revised correction terms for the third itinerary.

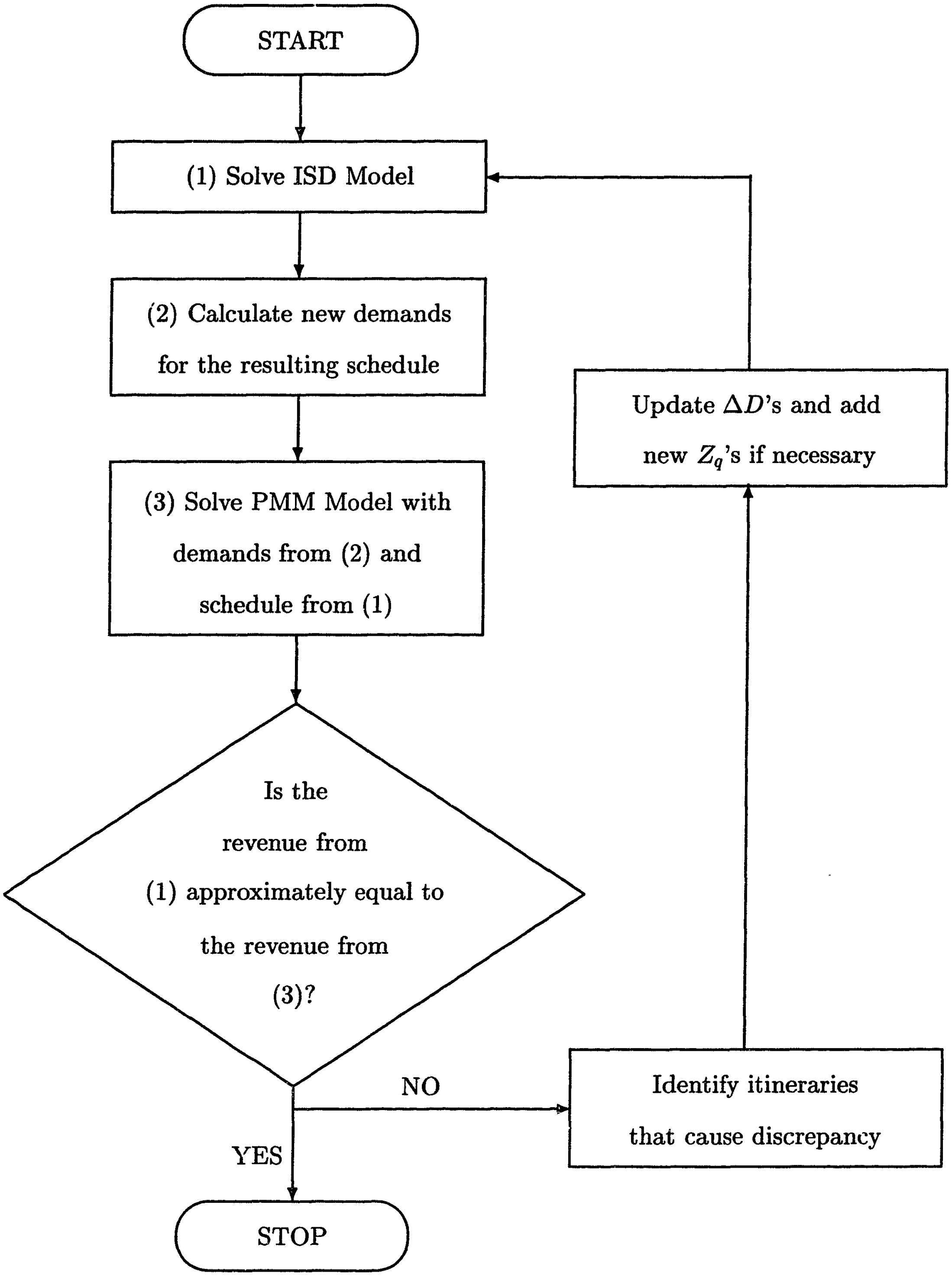


Figure 3-6: The Flowchart for Solution Approach II.

Table 3.2: Demand Correction Terms after the 1^{st} Iteration

Dropped Itinerary	Demand Correction Terms on Itinerary			
	1	2	3	4
3	0	40	N/A	20
4	5	38	15	N/A

Table 3.3: Demand Correction Terms after the 2^{nd} Iteration

Dropped Itinerary	Demand Correction Terms on Itinerary			
	1	2	3	4
3	7	40	N/A	18
4	5	38	15	N/A

Table 3.4: Demand Correction Terms after the 3^{rd} Iteration

Dropped Itinerary	Demand Correction Terms on Itinerary			
	1	2	3	4
3	7	40	N/A	18
4	5	38	15	N/A
3 and 4	-2	-28	N/A	N/A

At the beginning of the second iteration, the demand data for the full schedule is fed to the ISD model with the revised correction terms (Table 3.2). Assume that this time the model drops the third itinerary but not the fourth itinerary. The schedule evaluation model outputs the new unconstrained demands of 107, 190, and 118 for the first, second, and fourth itineraries, respectively. Again since these are first order corrections, they are exact. Table 3.3 summarizes the *exact first order correction terms* for both third and fourth itineraries.

Suppose that in the third iteration, the ISD model drops either one of the two open itineraries, we have the exact solution because after the first two iterations, we have exact corrections. If the ISD model drops both open itineraries, however, the correction terms become approximate. Suppose the schedule evaluation model outputs the unconstrained demands of 110 and 200 for the first and second itineraries, respectively, when both open itineraries are dropped. Table 3.4 summarizes the exact demand correction terms including a set of second order corrections for cancellation of both open itineraries. At this point we have complete and exact information for this market. (Recall, however, that our implementation does not include second or higher order correction terms.)

In the example described above, we are able to get exact values for all possible combinations. If, on the other hand, in the second iteration, the ISD model drops both itineraries 3 and 4, we will not be able to construct Table 3.3 (and therefore, Table 3.4) because we are not able to separate the *first order effect* of cancelling the third itinerary and the *second order effect* of cancelling both itineraries 3 and 4. The information from this iteration can be used to revise the correction terms only approximately.

3.3 Application of ISD Model to Fleet Planning

The objective of fleet planning is to identify the appropriate numbers and types of aircraft the airline should acquire in order to meet changing demands in the future. Good fleet planning is essential, in part, because the process of acquiring aircraft usually spans a period of a few years.

The ISD model can be used as a fleet planning tool by relaxing the count constraints (Equations 3.5 in ISD-A/D or 3.18 in ISD). By relaxing the count constraints, the model is free to use as many aircraft as it prefers. We can then identify the required number of aircraft of each type in the resulting schedules. Solving this relaxed problem on various potential future schedules would then identify the appropriate composition and size of the airline's fleet.

Chapter 4

Incremental Schedule Design without Flight Re-timings

4.1 Overview

In this chapter we present a case study that considers only flight additions and deletions. The intention is to provide a proof of concept of our approach. We use a small simple network. The resulting numbers shown are not representative of the impact of our approach on the full network problem faced by the airline. We present these results solely to provide some insights about solvability and potential impacts of our approach.

4.2 Case Study

We obtain the data set from a major U.S. airline. The implementation is in the C language on an HP 9000 model D370, 160 MHz workstation with an HPUX operating system version 10.20. The optimizer is CPLEX version 4.0.

4.2.1 Data

The case study for flight additions and deletions employs a simplified network with 13 airports/cities served by a major U.S. airline operating an hub-and-spoke network structure. The hub airports are Denver International (DEN), O'Hare International (ORD), and San Francisco International (SFO). There are 228 flights in total, 134 of them are fixed flights and 94 of them are open flights (Table 4.1). There are 55 aircraft available; the breakdown

Table 4.1: Number of Flights

Total Number of Flights	228	flights
fixed	134	
open	94	

Table 4.2: Number of Aircraft

Total Number of Aircraft	55	aircraft
AB320	25	
B737-500	8	
B757	12	
B77	10	

by fleet type is shown in Table 4.2. There are 140 origin-destination markets and 608 itineraries, 293 of which are fixed and 315 open.

4.2.2 Model and Solution Algorithm

Since we consider only flight additions and deletions in this case study, the ISD model reduces to the ISD-A/D model (Figure 3-3). The solution algorithm is the one described in section 3.2.4.

Tables 4.4 and 4.5 present the size of the constraint matrix. Table 4.4 shows the breakdown of columns by variable types and Table 4.5 shows the breakdown of rows by constraint types. Note that initially we relax all demand constraints (Equation 3.7). The ability to relax these constraints can have a tremendous impact because the number of itineraries grows exponentially as problem size increases.

Table 4.3: Number of Markets and Itineraries

Total Number of Markets	140	markets
Total Number of Itineraries	608	itineraries
fixed	293	
open	315	

Table 4.4: Number of Variables

Total Number of Columns	2368	columns
$f_{k,i}$	912	
$y_{k,o,t}$	534	
t_r^p	607	
Z_q	315	

Table 4.5: Number of Constraints

Total Number of Rows	2302	rows
Cover constraints (Equations 3.2 - 3.3)	228	
Conservation of flows constraints (Equation 3.4)	534	
Capacity constraints (Equation 3.6)	228	
Demand constraints (Equation 3.7)	608	
Itinerary status constraints (type I) (Equation 3.8)	385	
Itinerary status constraints (type II) (Equation 3.9)	315	

4.2.3 Results and Analysis

In this section, all of the results are compared to the results from Keypath-ODFAM (Figure 2-3) performed on the network containing all of the open flights in the schedule; that is, all of the open flights are assumed as fixed and have to be flown.

The solution times for the ODFAM and ISD-A/D models are 8 and 113 seconds, respectively. The lower bound on the objective function value for ISD-A/D model is obtained from the LP relaxation and the optimality gap is within 1%. The model generates 78 rows out of the 608 rows initially relaxed (13%) in its LP relaxation.

The costs are reported in Table 4.6. We notice that spill costs are increased in ISD-A/D model, more so than in ODFAM (almost 26 times). This means that we indeed are spilling passengers almost everywhere in the network. The benefit of ISD- A/D, however, derives from the savings from carrying costs and operating costs. The carrying costs are the costs incurred for each passenger carried, including baggage handling, meal, reservation fee (paid to travel agent, and Computerized Reservation System (CRS) owner), etc. The operating costs are flight operating costs. A significant amount of savings comes from flight cancellations. Table 4.7 shows that 70 open flights (75% of open flights or 31% of total flights) are dropped out of the schedule; this leads to savings in operating cost of $351,210 per day. In total, we are able to save $37,086 per day. We remark again that these numbers

Table 4.6: Costs and Savings‡

Costs	ODFAM	ISD-A/D
Total cost	2,453,753	2,416,667
Saving		37,086
Spill cost	18,921	490,421
Saving		-471,500
Carrying cost	901,805	744,428
Saving		157,377
Operating cost	1,533,027	1,181,817
Saving		351,210

‡Dollars per day

Table 4.7: Number of Open Flights Covered

Total Number of Open Flights	94	flights
covered	24	
dropped	70	

serve only as proof of concept purpose and do not indicate any level of potential savings for the real-sized problem.

It is very interesting to note that out of 24 open flights that are selected in the resulting schedule, 22 of them are flights in the DEN-SFO and SFO-DEN markets (11 in each direction) and 2 of them are flights in the ORD-SFO and SFO-ORD markets(1 in each direction). These selected open flights are flights between hub airports. Tables 4.8 - 4.10 show the flights between three hub airports in our network, including breakdowns of fixed and open flights. We direct our focus to Table 4.10, which shows that 11 flights out of 12 flights in this link are open flights and the final schedule includes all of these open flights. The explanation is that these flights between DEN-SFO and SFO-DEN are very high yield flights. Table 4.11 indicates that on average the SFO-DEN (DEN-SFO) link carries passengers from 8 (11) different markets. In extreme cases, 22 markets can be served by flights from DEN-SFO. This is not surprising. Hub-to-hub flights typically have very high load factors because they consolidate demands from many different markets at both ends of the link.

Open flights flying between spoke and hub airports (in this network, in particular), on the other hand, carry not as many markets, and therefore, do not yield high revenue. The

Table 4.8: Flights in ORD-DEN and DEN-ORD

Number of Flights (each direction)	11	flights
fixed	9	
open	2	
open and flown in the resulting schedule	0	

Table 4.9: Flights in ORD-SFO and SFO-ORD

Number of Flights (each direction)	10	flights
fixed	6	
open	4	
open and flown in the resulting schedule	1	

Table 4.10: Flights in DEN-SFO and SFO-DEN

Number of Flights (each direction)	12	flights
fixed	1	
open	11	
open and flown in the resulting schedule	11	

Table 4.11: Number of Market Served on Selected Links

Link	Maximum	Average	Minimum
SFO-ORD	2	2	2
ORD-SFO	9	9	9
SFO-DEN	15	8	3
DEN-SFO	22	11	3

model thus drops them out of the schedule. This explains why we see very high spill costs in the resulting schedule. It is important to note that we see such low yield rates because our network is a reduced network that contains only 13 cities—10 spoke cities, in particular. In the full network, those spoke-to-hub flights carry passengers from many more markets, and thus, yield much higher revenues.

4.3 Summary

In this chapter we present a case study serving as a proof of concept for our incremental schedule design model that considers flight additions and deletions. We have seen that our ISD-A/D model correctly selects high yield flights between hub airports and drops low yield flights between hub and spoke airports. It is important to note that this result is specific to our network containing 3 hub airports and 10 spoke airports, as discussed in Section 4.2.3. We have seen that the savings from ISD-A/D models derive mainly from savings in operating costs, resulting from flight cancellations. Flight cancellations, on the other hand, cause higher spill costs. Nonetheless, in total, the savings are large enough to offset the higher spill costs.

In chapter 5, we consider the incremental schedule design problem for flight re-timings only. We present different operating scenarios and show their effects on incremental scheduling.

Chapter 5

Flight Scheduling in the Context of Free Flight

5.1 Overview

It is estimated that the air traffic growth rate will be around 3 to 5 percent for at least the next 15 years [19]. This will exacerbate congestion problems in the already congested National Airspace System (NAS). Efforts have been made to devise air traffic control (ATC) systems employing new technologies that enhance the efficiency of NAS without compromising user safety. One such idea is known among air transportation professionals as *Free Flight.*

Free Flight is a new concept designed to alleviate increasing congestion in the air space. The idea is to move the current centralized command-and-control system between pilots and air traffic controllers to a distributed system that allows pilots, whenever practical, to choose their own route and file a flight plan that follows *the most efficient and economical route* [19]. Free Flight requires, among other things, use of state-of-the-art technologies in communication in order to provide users and service providers real-time and accurate information.

Since Free Flight allows more direct routing, an airline benefits in two ways:

1. flight block times are reduced; and/or

2. flight schedule reliability is increased.

In our analysis we focus on the first benefit because it has direct impacts on scheduling.

It is obvious that reduced block times achieve savings due to reduced fuel burn. Moreover, further savings can be achieved by exploiting resulting scheduling opportunities. In this case study, we consider only the effect of flight re- timing and assume constant demand and a fixed flight set (that is, no flights are added or deleted from the schedule). In future research, we can relax these assumptions.

5.2 Changes to the ISD Model

In simplifying our evaluation of the impact of Free Flight on the flight schedule, we do not consider the opportunity for flight addition and/or deletion. The ISD model, therefore, reduces to the FAMTW model. The critical input data for this application is the revised block times for each flight. It is not clear how much reduction in block times can be achieved with Free Flight. In our case study, we use an average reduction of 10% in block times across the network.

5.3 Illustration

We illustrate in Figure 5-1 how flight re-timings can improve the schedule in the context of Free Flight. There are three airports (BOS, ORD, and SFO) and two flights (A and B) in this small example. Flights A and B are the originally scheduled flights from BOS to ORD and from ORD to SFO, respectively. Flights A′ and B′ are the newly scheduled flights, resulting from block time reductions. The small boxes surrounding the departure and arrival times represent allowable time windows within which the flights can be re-timed.

We can see that initially with only flights A and B in the schedule we cannot connect A to B even with re-timing allowed. This implies that:

1. the through revenues that would have been generated if the two flights could have been flown with the same aircraft are lost; and

2. higher costs might be incurred because two aircraft, rather than one, are needed to fly these two flights.

With reduced block times, flight A′ can be moved downward along the time line and flight B′ can be moved upward along the time line so that the two flights can be flown with the same aircraft. Such re-timings through out the schedule can have significant impacts

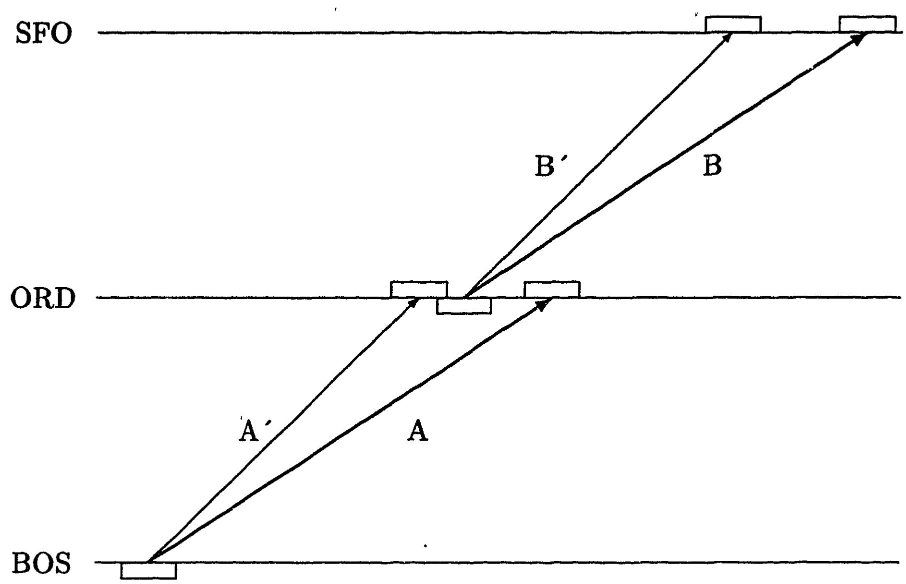


Figure 5-1: Effect of Block Time Reduction on Scheduling (not to scale)

Table 5.1: Data Set

Problem	Number of Fleet	Number of Flight	Number of Aircraft
P1	2	456	112
P2	3	1445	299
P3	7	2037	432

since they allow more connecting opportunities and tighten up the schedule potentially allowing the schedule to be flown with fewer aircraft. The advantages are that aircraft are more efficiently utilized and, the aircraft that are not utilized in the new schedule can be sold, leased to others, or used to open new markets or increase frequencies in existing markets.

5.4 Case Study

5.4.1 Data

Our Free Flight case study uses data (Table 5.1) from a major U.S. airline. Problem P3 is the full network with 7 fleet types. Problems P1 and P2 represent smaller versions of the full network.

1. Aggregate the matrix using optimizer's algebraic preprocessor
2. Perturb problem using the optimizer's perturbation scheme
3. Solve the LP using dual steepest-edge simplex
4. Remove perturbations
5. Reoptimize using dual steepest-edge simplex
6. Disaggregate the matrix
7. Reoptimize using dual steepest-edge simplex
8. Prioritize special ordered sets
9. Solve MIP using branch-and-bound (B&B)

Figure 5-2: FAMTW Solution Algorithm

5.4.2 Solution Algorithm and Implementation

We evaluate the impact of Free Flight on the flight schedule and its fleeting using the FAMTW IST algorithm of Rexing, et al. [29]. The algorithm is implemented in the C language on an IBM RS/6000, Model 370 workstation, 256MB RAM with the CPLEX callable library version 3.0 [12]. Figure 5-2 outlines the solution algorithm of Rexing, et al. [29].

In Step 1, the problem size is reduced using the algebraic preprocessor in CPLEX. In Step 2, the variable bounds are perturbed in order to reduce the degree of degeneracy and thus, improve the performance of the simplex algorithm. The LP of the FAMTW is solved in Step 3 using the dual steepest-edge simplex, the method recommended by Hane et al. [21] for solving FAM. In Steps 4- 7, the solution of the LP relaxation is obtained by removing perturbations, resolving, and disaggregating the problem. In Step 8, special ordered sets (SOS) of the cover constraints are identified and prioritized. These SOS are groups of binary variables of which only one can have a nonzero value. Step 9 will use these SOS as branching decisions in the branch-and-bound tree. The sets are constructed by ordering the variables in non-decreasing aircraft capacity. They are then divided into 2 subsets such that half (or close to half) of the variable fractionality lies in one subset and half in the other subset. Branching is performed on a subset. Branching priority is given to the set with the largest objective coefficient gap between the first and last variables of that set.

Table 5.2: Description of the Scenarios

Scenario Name	Description
SC0	Traditional FAM model
SC1	FAMTW model
SC2	FAMTW model with reduced block times (RBT)
SC3	FAMTW model with RBT and secondary objective of minimizing number of aircraft

5.4.3 Scenario Descriptions

We experiment with several scenarios shown in Table 5.2. Scenario SC0, the FAM model with fixed departure times and blocktimes in an operating environment without Free Flight, is the *base case* against which we will evaluate the results from other scenarios. Scenario SC1 is the FAMTW model with time window width of plus/minus 20 minutes and copy interval of 5 minutes. Block times again reflect flying times prior to Free Flight. The purpose of SC1 is to illustrate the benefits of time windows. Scenario SC2 is the FAMTW model with reduced block times reflecting Free Flight. Scenario SC3 adds to scenario SC2 a secondary objective of minimizing the number of aircraft required. To minimize the number of aircraft required, we add a cost of flying each aircraft to the objective function. Equation 5.1 shows the modified objective funtion. $\delta_{n,k,i}$ equals 1 if $f_{n,k,i}$ is an element of $CL(k)$, and 0 otherwise. C_k is the cost of aircraft of fleet type k.

$$Min \sum_{i \in L} \sum_{k \in K} \sum_{n \in N_{ki}} (C_{n,k,i} + \delta_{n,k,i} C_k) f_{n,k,i} + \sum_{k \in K} \sum_{o \in O} y_{k,o,t_n} C_k \tag{5.1}$$

5.4.4 Results

The running times for these problems and scenarios are reported in Table 5.3. The running times for smaller problems (P1 and P2) are very short. Most of them are under 3 minutes except only for problem P2 under scenario SC3 where the running time increased to 14 minutes. Notice also that the running times for scenario SC2 are less than that of scenario SC1 for problem P1 and P2. This is a result of the less-tight, reduced-block-time schedule. This is not the case, however, when the problem becomes larger, as in problem P3, where

Table 5.3: Running Time† of FAMTW model

Problem	SC0	SC1	SC2	SC3
P1	0.03	0.13	0.12	0.12
P2	0.33	2.43	1.80	14.20
P3	10	76.35	*	29 hours

†Running Times in minutes except where noted
*Did not reach solution due to insufficient memory

Table 5.4: Total Costs and Savings‡

Problem		SC0	SC1	SC2	SC3
P1	Total Cost	8,234,834	8,213,515	8,201,902	8,206,472
	Saving		21,319	32,932	28,362(+)
P2	Total Cost	20,151,734	20,115,656	20,083,927	20,106,817
	Savings		36,078	67,807	44,917(+)
P3	Total Cost	28,958,190	28,888,619	*	28,905,716
	Saving		69,571	N/A	52,474(+)

‡Dollars per day
* Did not reach solution due to insufficient memory

we experience difficulties in solving the problem. Scenario SC2 for problem P3 is not solved due to insufficient memory. We are able to solve scenario SC3 for problem P3, but the solution time is long.

Table 5.4 summarizes the total costs for each problem and scenario over the base case and the associated savings in dollars per day. There are two components to the cost presented here. The first component is the direct flight operating cost. The second component is the spill cost. We can see that FAM produces the highest cost assignments because it uses full block times that do not reflect Free Flight and it does not allow flight re-timings. The cost of scenario SC2 is less than that of scenario SC1 because of the reduction in block times. Just to emphasize the benefit, the savings of $69,571 per day in problem P3 under scenario

Table 5.5: Reduced number of Aircraft Required under Scenario SC3

Problem	Reduced Number of Aircraft Required
P1	1
P2	5
P3	17

Table 5.6: Effects on Scheduling for Problem P1

	SC1	SC2	SC3
Number of flights re-fleeted	37	56	54
	(8.11%)	(12.28%)	(11.84%)
Number of flights re-timed	21	45	42
	(4.61%)	(9.87%)	(9.21%)
Average amount of re-timing (abs mins)	16.52	12.60	12.64

Table 5.7: Effects on Scheduling for Problem P2

	SC1	SC2	SC3
Number of flights re-fleeted	129	178	167
	(8.93%)	(12.32%)	(11.56%)
Number of flights re-timed	76	121	137
	(5.26%)	(8.37%)	(9.48%)
Average amount of re-timing (abs mins)	15.76	12.99	13.81

SC1 translates to approximately $25 million per year. We use the (+) notation beside the savings of scenario SC3 to highlight the fact that savings from reducing the number of aircraft required are not included (Table 5.5). Total savings are certainly higher than the numbers reported since the total saving would include cost reductions from not flying these aircraft and potential profits from utilizing these aircraft elsewhere.

The stopping condition for terminating the algorithm is when the difference between the objective function value of the LP relaxation and that of the integer programming is less than or equal to $3,000 per day. Therefore, the resulting optimality gap for problems P1 - P3 are within 0.04%, 0.015% and 0.010%, respectively.

Tables 5.6 to 5.8 show the effects of Free Flight on scheduling for problems P1 - P3,

Table 5.8: Effects on Scheduling for Problem P3

	SC1	SC2	SC3
Number of flights re-fleeted	301	N/A	393
	(14.78%)	(N/A)	(19.29%)
Number of flights re-timed	21	N/A	324
	(7.90%)	(N/A)	(15.91%)
Average amount of re-timing (abs mins)	15.27	N/A	13.23

respectively. All of the comparisons are relative to scenario SC0. We notice similar effects in problems P1 and P2, and more pronounced effects for problem P3. The number of re-fleeting and re- timings in problem P3 are increased to reflect that to minimize the number of aircraft used (17 fewer aircraft are necessary for SC3), a number of schedule and fleeting changes are necessary.

5.5 Summary

We show that with new operating policies, such as Free Flight, improvements in flight schedules can be achieved using our ISD model. We present detailed effects of block time reductions on flight schedules and their fleeting. We show that with block time reductions of 10%, a new schedule can be produced with annual estimated savings of approximately $20M. This estimate of the impact of reduction in block times does not include the potential profits from utilizing the freed-up aircraft, neither does it consider the effects of flight additions and/or deletions.

Chapter 6

Conclusion and Future Research

6.1 Conclusion

The objective of this thesis was to devise tools for incremental schedule design. The potential changes that can be made to the schedule are:

1. flight re-timings, and
2. flight additions and deletions.

We have shown that the FAMTW model can be used for incremental scheduling allowing flight re-timings and the ISD-A/D model can be used for incremental scheduling allowing flight additions and deletions. The ISD-A/D model is described in Chapter 3. We emphasize here again that the validity of the resulting schedule from the ISD-A/D model relies heavily on the accuracy of the demand correction terms. We have developed a modeling framework, formulation, and solution algorithm for solving incremental schedule design problems that simultaneously allow both flight re-timings and flight additions and deletions. Additionally, our approach integrates part of the schedule design step with the fleet assignment step (Figure 1-1).

Due to the size and complexity of the problem, we present two separate case studies: one that addresses only flight additions and deletions (Chapter 4) and the other addresses flight re- timings (Chapter 5).

The first case study serves as a proof of concept of our approach. The model is run on a simplified network containing only 13 airports. We observe that the model correctly selects high yield, hub-to-hub flights and drops low yield, spoke-to-hub (and hub-to-spoke) flights.

We have discussed that this result is specific to this data set and the figures shown do not represent the level of savings that could be achieved in the full-sized network.

In the second case study, we compare several operating scenarios using different data sets, including the full network with 2,037 flights. We try to quantify the benefits of Free Flight. We have seen that with a 10% reduction in block times (as a result of Free Flight) an approximate annual savings of $20M can be achieved. In another scenario, we have seen that by allowing re- timings and reductions in block times, the schedule can be flown with 17 fewer aircraft. These freed-up aircraft yield significant savings.

6.2 Future Research

This thesis represents the first steps of research in the area of airline schedule design. There are many research questions left unanswered or unexplored. We categorize them as follows.

6.2.1 Short and Medium Term Research Directions

In the short term, analysis of the model to measure the sensitivity of the demand correction terms must be performed. The information from such analysis will allow us to fine-tune the model. Specifically, if the model turns out to be relatively insensitive to demand correction terms, this might allow us to safely use approximations of these terms. On the other hand, if the model is sensitive to these terms, this might indicate that we need higher degree correction terms for cases where two or more open itineraries interact in a market.

In the medium term, a careful study of the demand and supply interactions in the schedule can be helpful. It may reveal some useful trends that can be used to more accurately approximate the demand correction terms.

6.2.2 Longer Term Research Directions

In the longer term, the following are potential research directions.

1. **Apply the ISD-A/D model to the full network.** This requires a better understanding of model behavior. Exploitation of network structure might be necessary. A new solution algorithm must be devised in order to allow the model to handle prohibitive problem sizes.

2. **Search for alternative approaches.** Instead of focusing solely on the simultaneous, exact approach, we should allow ourselves to explore decomposition and heuristic approaches that might be able to facilitate the problem's solution considerably without severely compromising optimality.

3. **Enable the model to propose potential additions and deletions.** Currently, we take the list of open flights as given. Such lists can be constructed by experienced planners. However, enabling the model to create these lists could be very helpful, since computers can do more exhaustive searches.

4. **Increase the degree of demand correction term.** This may be required if sensitivity analysis indicates that the model is highly sensitive to these terms. It will require a more complex solution algorithm since the problem size could grow substantially when we increase the degree of the correction terms.

5. **Implement the simultaneous model.** The simultaneous ISD model that allows for both flight re-timings and flight additions and deletions at the same time could provide impressive economic results. However, the real benefit of this can only be realized after we are able to solve the ISD-A/D model on the full-sized network.

6. **Adopt more aggregate demand data.** Currently, we are using unconstrained demand data at the itinerary level. These are small fractional numbers causing accuracy and precision problems. Developing an alternate model that is based on market demand data can be advantageous in many ways. First, we can avoid the accuracy and precision problems of the data. Second, more aggregate demand data might lead to smalier constraint matrixes. Third, removing restrictions at lower levels might lead to better quality solutions.

Appendix A

Glossary

A.1 Definitions

aircraft day: a series of connecting flights that is flyable by a single aircraft in one day.

aircraft rotation: the sequence of flight legs that are flown by an aircraft beginning and ending at the same station.

aircraft routing: the process which determines the actual aircraft rotation in a fleeted schedule, the third step in the airline schedule planning process.

airline schedule planning process: the four-step sequential planning process consisting of (1) schedule design, (2) fleet assignment, (3) aircraft routing, and (4) crew scheduling.

connecting bank or complex: a set of flights arriving or departing a hub airport in some period of time.

crew pairing: a sequence of connected flight segments that begins at a crew base location, and returns to the same location, within the maximum allowable *time-away-from-base*. A pairing is a set of *duty periods*, separated by *rest periods/overnights* that satisfy all work rules.

daily flight schedule: a flight schedule that repeats every day of the week.

deadhead: a flight leg that has one or more crews flying a passengers. Deadheading is used to transport a crew from one station to a target station, in order to utilize the crews (by assigning them to fly flights deparing the target station) more effectively.

domestic schedule: the flight schedule orginating with flights and terminating within the United States.

duty period: a sequence of connected flights legs that can be flown legally by a crew in one day.

fleet assignment: the process of assigning fleet types to flight legs. It is the second step in the airline schedule planning process.

flight copies: copies of a flight with different departure times.

flight crews: pilots or flight attendants.

fleeted schedule: a flight schedule with eery flight in the schedule assigned to a fleet type.

flight schedule: the list of flights with specific origin and destination locations, and departure and arrival times for each flight.

itinerary: a sequence of flights from originating city to the destination city.

maintenance station: the station (airport) that is capable of performing aircraft maintenance.

rest period: the non-work period between two duty periods.

schedule design: the selection of flight legs and their schedules to be flown by the airline.

It is the first step in the airline schedule planning process.

spilled passengers: passengers that cannot be accommodated due to capacity constraints.

spill cost: the amount of revenue that is lost to the airline due to inability to accomodate the total demand as a result of fleeting decision.

time-away-from-base: the total duration of a pairing, that is, the difference between the arrival time of the last flight leg and the departure time of the first flight leg in the pairing, plus the brief time for the first flight leg and debrief time for the last flight leg [24].

unconstrained demand: the highest attainable demand level for the airline (or total number of requests).

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